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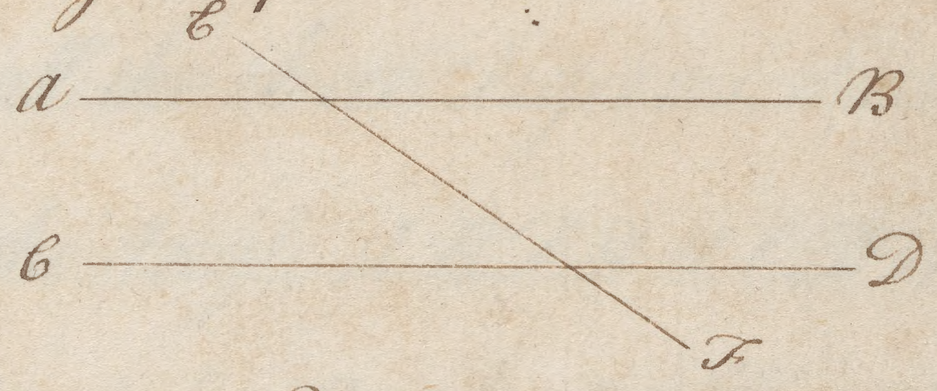
Geometrical Theorems

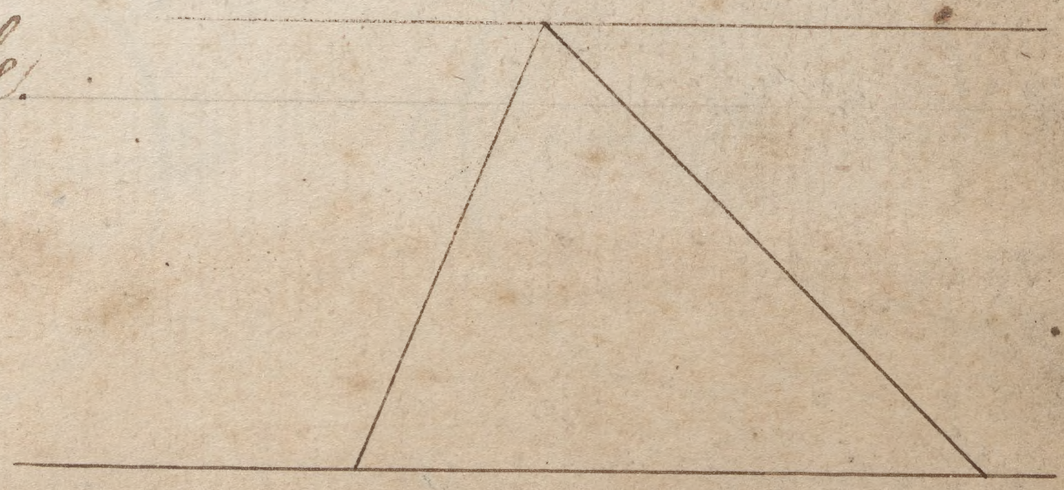
1

If two right Lines be parallel at any Distance as A B & C D & be cut by a third E F at any Angle whatsoever the Angles of the one are equal the Angles of the other.



2

A Rectilineal Triangle made between two parallel Lines the internal Angle is equal to the external; & that all the three Angles taken together are equal to a Semicircle.



MSS 839B
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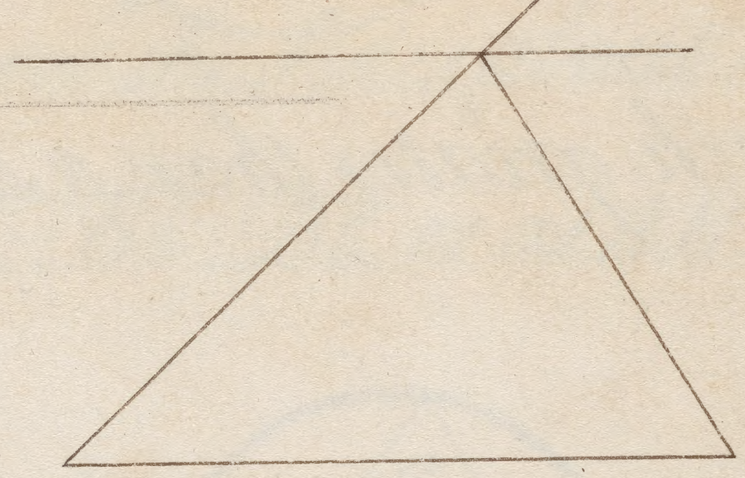
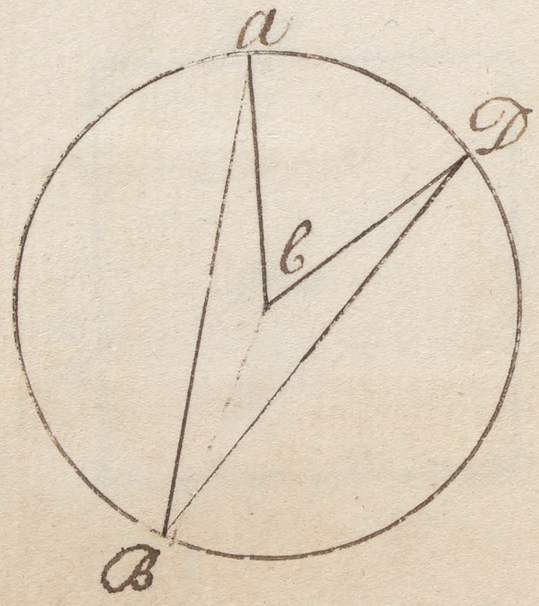
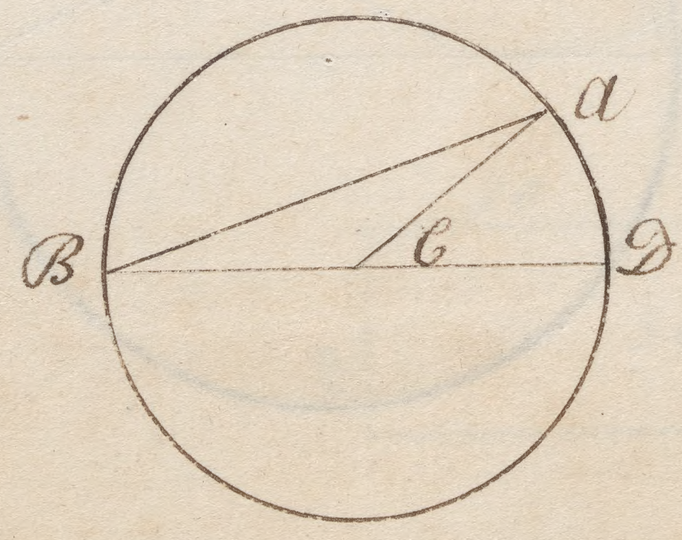
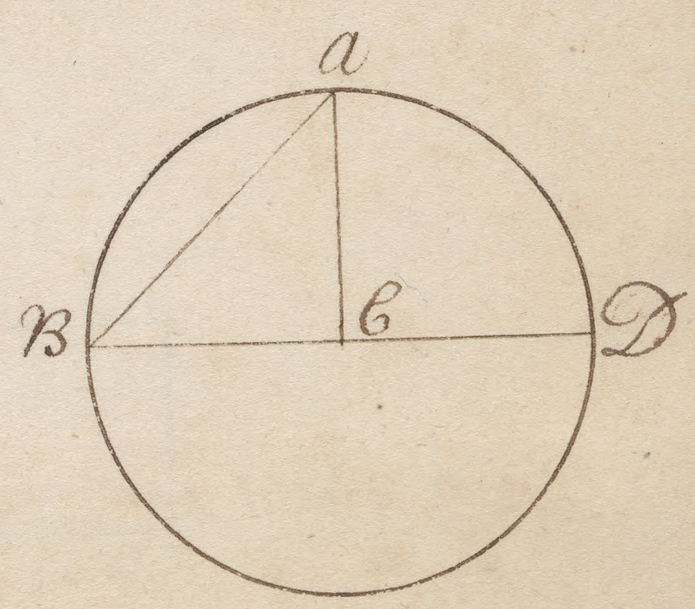
any Side of a rectilineal Triangle being produced the external An-
gle is equal to the Sum of the two acute internal Angles. And farther,
if we draw a line parallel to the Base it will divide the external Angle
into two other Angles which will be in Quantity equal to the two inter-
nal Angles separately.



An Angle at the Center is double to an Angle at the Circumference,
as is evident from all the three Figures; the Angle A C D in the first
Scheme is double to the Angle A B D. & a C D in the second is double to
the Angle a B D. And the Angle a C D in the third Figure is dou-
ble to the Angle a B D.





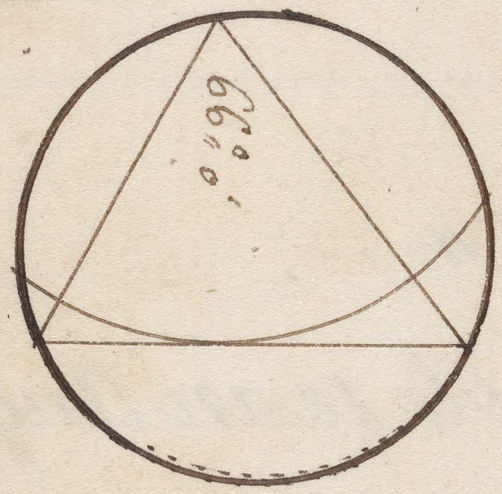


5

Every equilateral Triangle inscribed in a Circle the internal Angles in the Triangle are equal to a Semicircle & the external to the whole Circle or 360.°

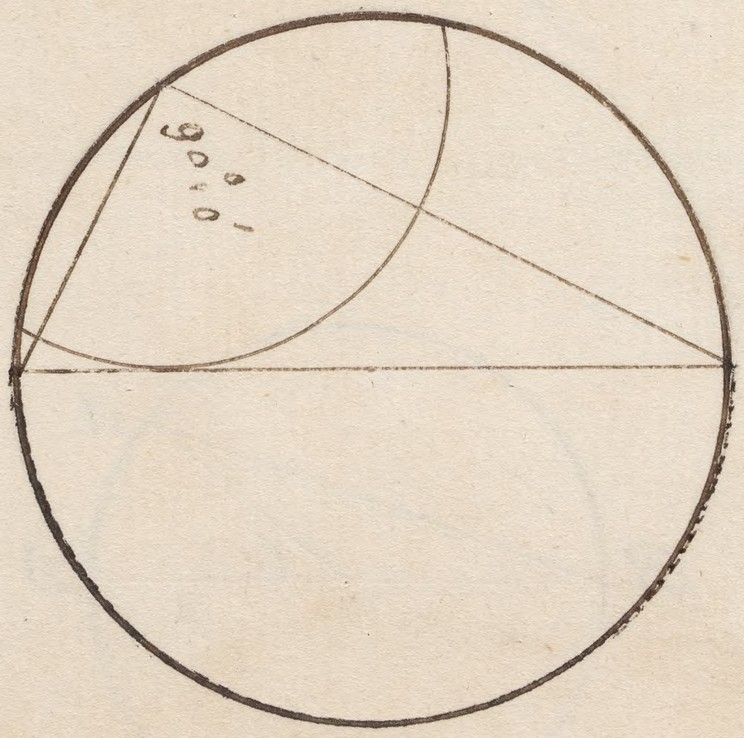
6

An Angle in an Arch greater than a Semicircle is less than a Quadrant or 90.°



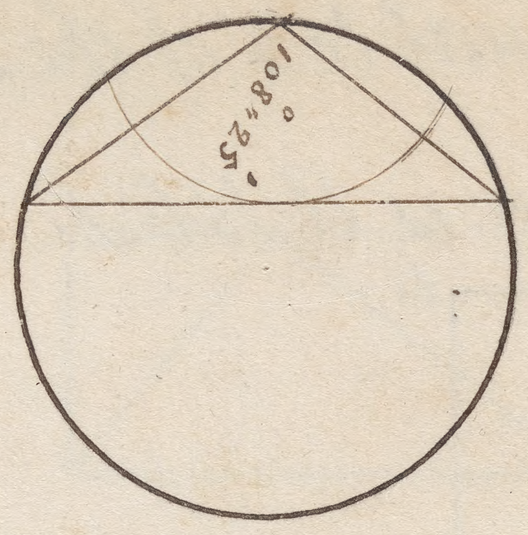
7

An Angle in a Semicircle is equal to a Quadrant or 90.°



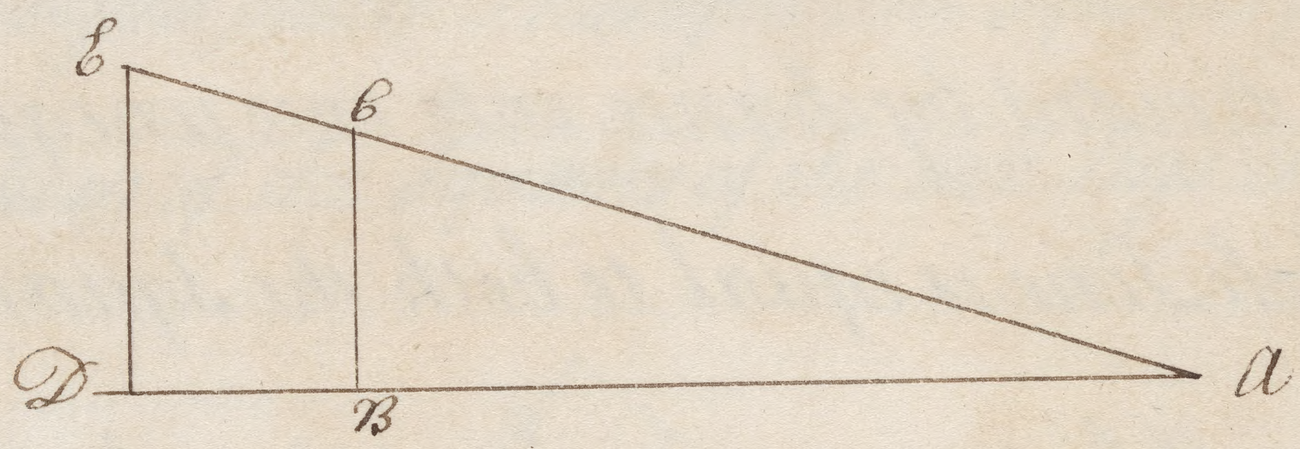
8

An Angle in an Arch ~~greater~~ less than a Semicircle is greater than a Quadrant or 90.°



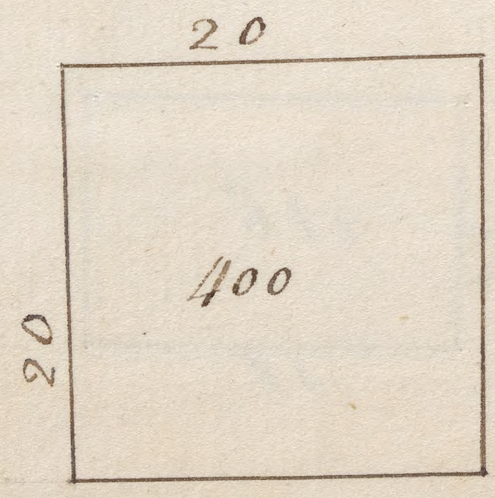
9

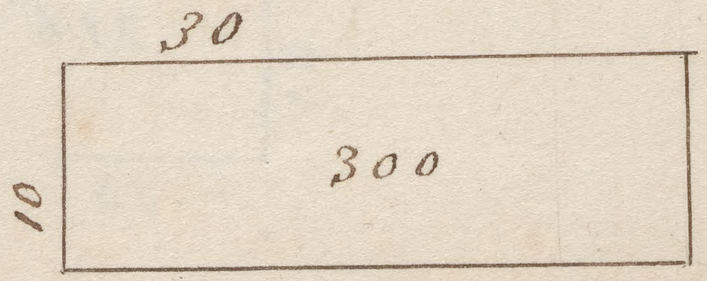
Equiangled Triangles have the Sides about the equal Angles proportional. for as AB: AD:: Bc: Dc, or as AB: Bc:: AD: Dc.



10

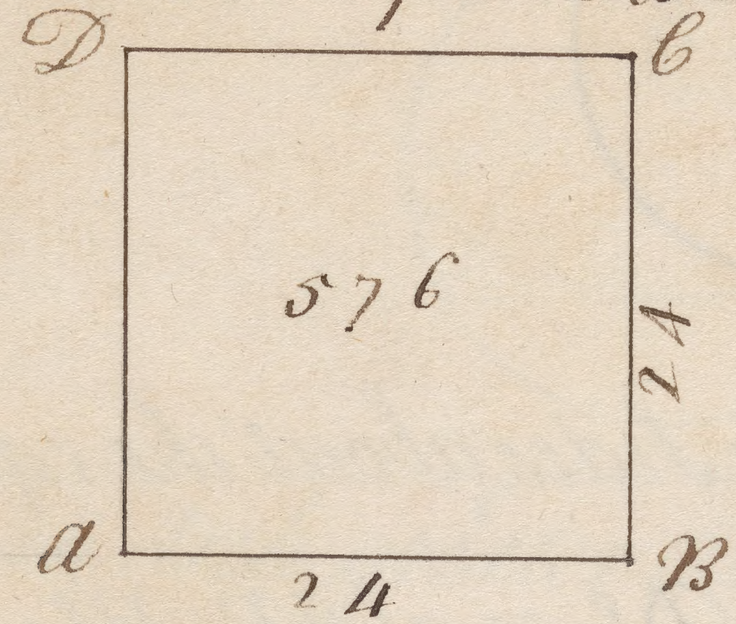
The Area of superficial Figures are proportional as the Square of their Sides that bound them for suppose the Side of a Square 20 & the Length & Breadth of a parallelogram 30 & 10 the Area of the Square exceeds that of the parallelogram proportional to the Square of their Sides.

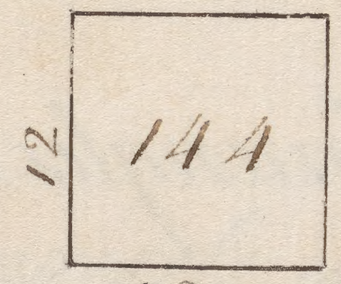




11

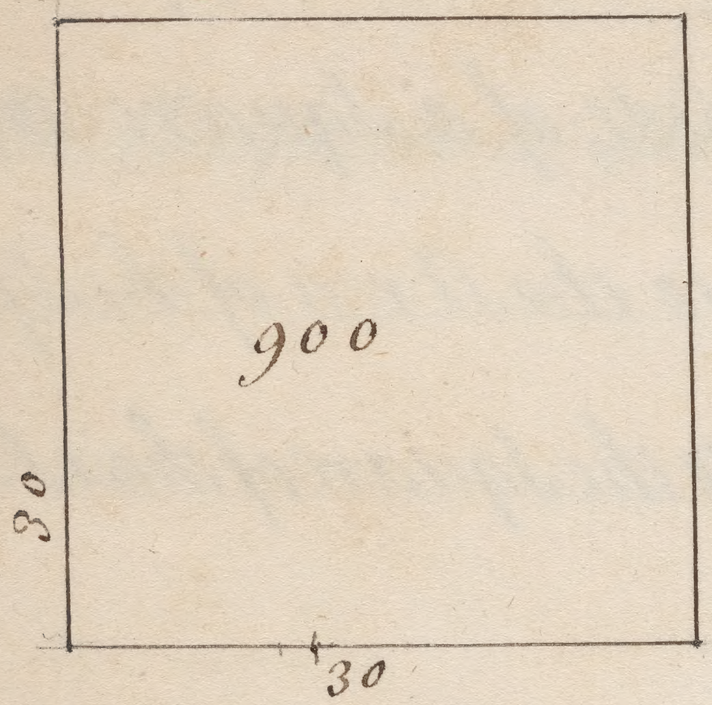
When a Right Line is divided into two equal parts the Square made of one of those parts is equal to ¼ of the Square made of the whole Line AB whose square is ABCD.

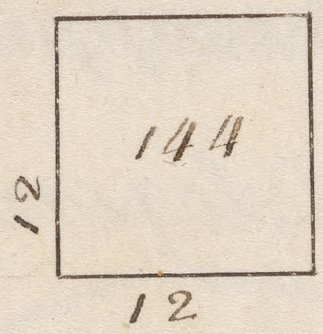


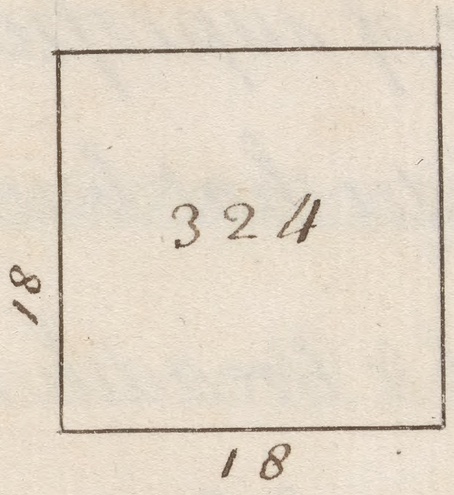


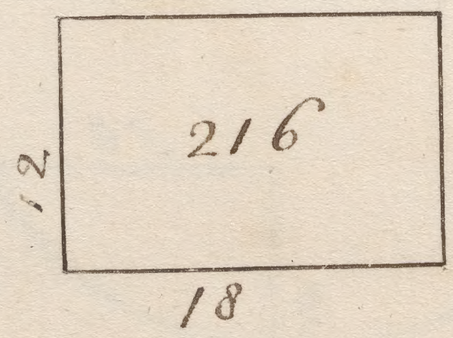
12

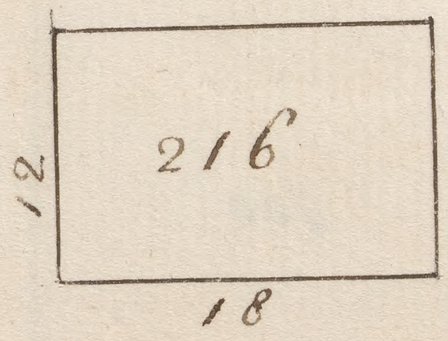
When a Line is divided by chance into two unequal parts the Square of the whole Line is equal to both the Squares made of the parts & to two Parallelograms comprehended under the same parts also.



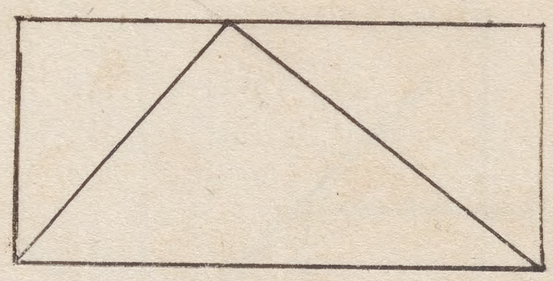




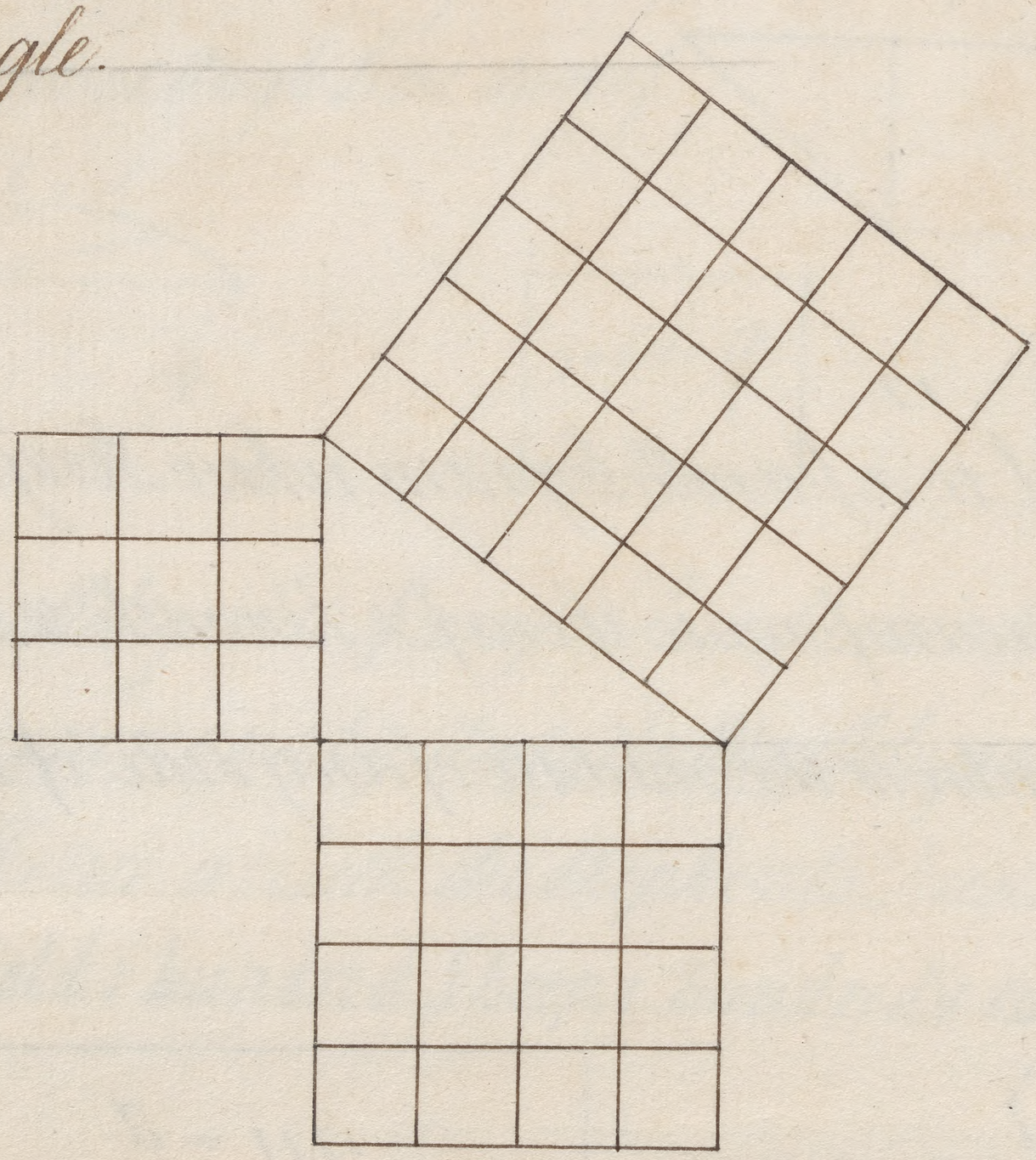




Every rectilineal Triangle is half the Square or Oblong that cir=
=cumscribes it.

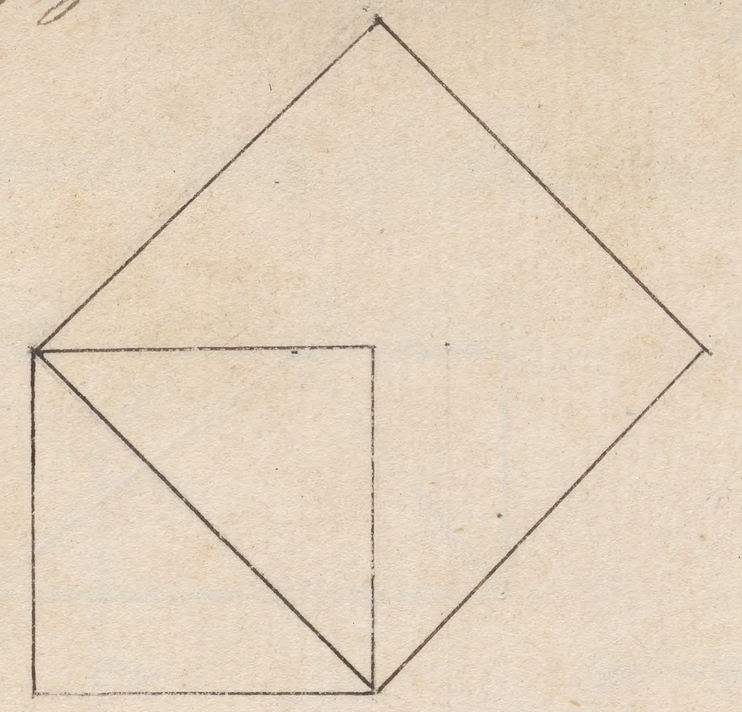


In all right Angled plain Triangles The Square of the Hypothenuse is equal to the Sum of the Squares of the two Legs containing the Right Angle.



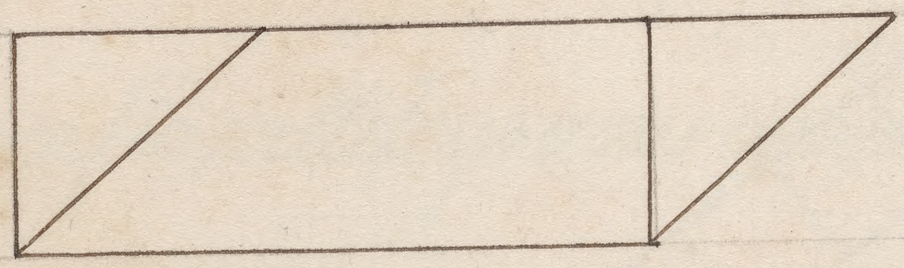
15

The Diagonal Line of every Geometric Square is double in power to the Side of the same Square.



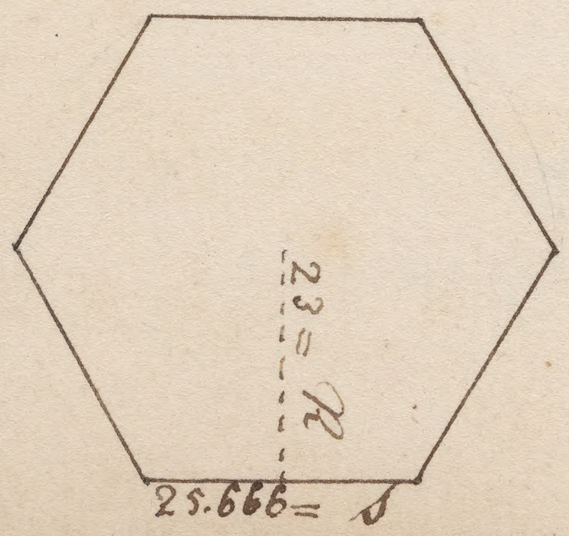
16

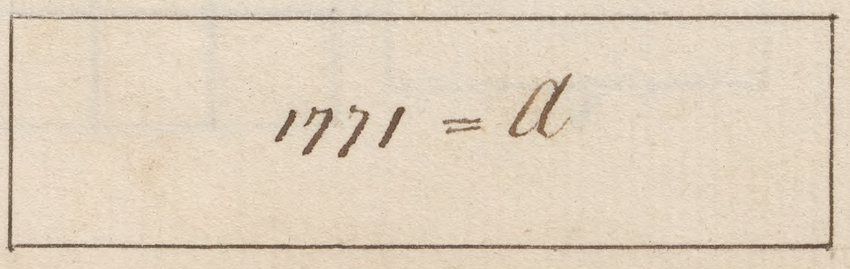
Parallelograms which stand on the same Base or on equal Bases & in equal or the same parallels are equal one to the other.



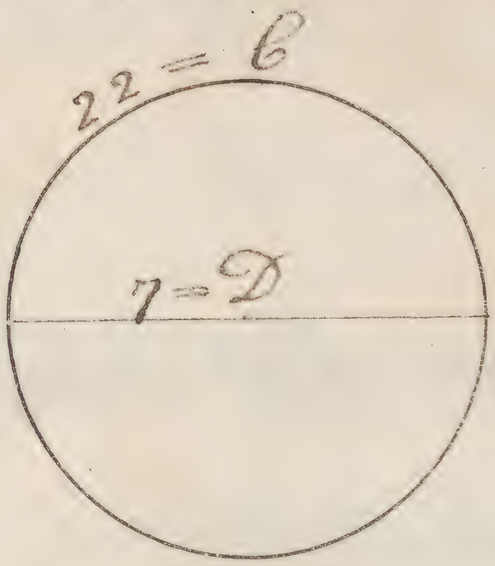
17

Every Polygon is equal to a Parallelogram whose Length is equal to half the Perimeter or Circumference thereof & Breadth to a Perpendi=cular drawn from the Centre to the middle of any side of the same.



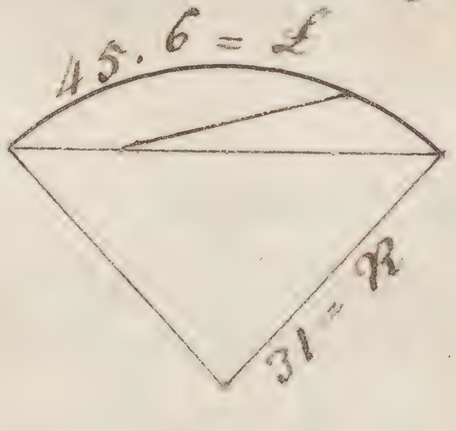


Every circle is nearly equal to a parallelogram whose length is equal to half the Circumference & Breadth to the Semidiameter.



38.5 = a

Every Circles Sector is nearly equal to a parallelogram whose Length is equal to the Semidiameter of the Circle & Breadth to half the Curve or Arch Line thereof. Or whose length is equal to half the Semidiameter & Breadth the whole Curve or Angle Line thereof.



706.8 = a

706.8 = a

If any parallelogram of any Dimensions whatsoever has for its end Unity if each end be encreased by Unity the parallelogram will then be double to what it was before. Hence ariseth the Shepherds Question Viz If a Hundred Hurdles will fold 1 hundred Sheep, 1 hundred & two Hurdles will fold 200 Sheep.

Explanation of Terms.

1 Axiom, is a Principle in any Art so evident that it needs no=
thing but the Light of Reason to demonstrate it

2 Construction, is the drawing of Lines & framing of Figures on pre=
paring the Proposition for a Demonstration.

3 Corollary is a consequent Truth gained from preceding
Demonstration.

4 Definition is the unfolding or explicating of the Nature &
Affection of a Thing in a few Words.

5 Demonstration is the proving a thing by Definitions & Axioms
& so from several Arguments drawing a Conclusion that
it has that Affection the Proposition did assert.

6 Hypothesis is when a Thing is supposed or given
to be so.

7 Lemma is the Demonstration of some Premise in Order
to shorten a following Demonstration.

8 Problem is when something is propos'd to be done.

9 Proposition is used promiscuously either for a Problem
or Theorem.

10 Postulata is a grantable Request or such a Demand as
cannot reasonably be denied.

11 Scholium is a short or critical Expasition gained from a for-
mer Demonstration, or a Corollary wanting an Explanation

12 Theorem is when something is proposed to be demonstrated.

Geometrical Problems

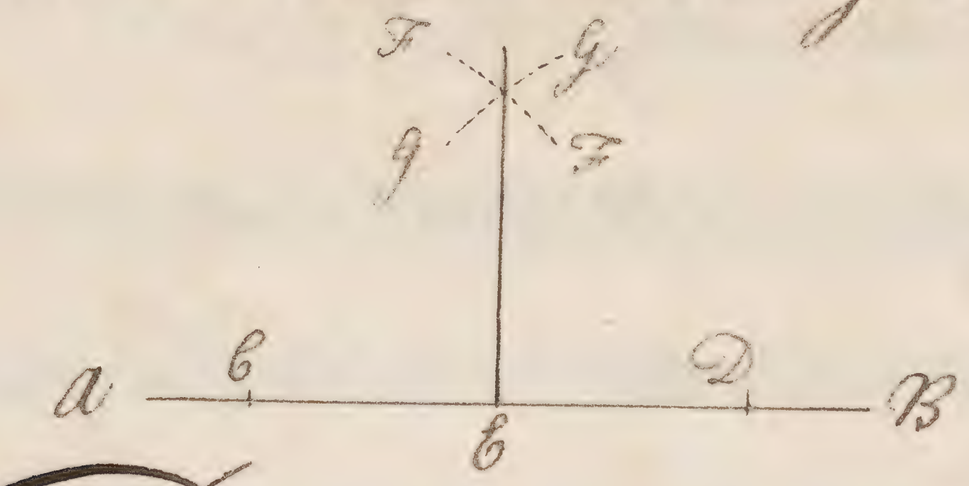
Problem 1.

Upon a right Line given how to erect another right Line which shall be perpendicular to the former.

Let the right Line A B be given upon which from the point
C erect a Perpendicular.

Open the Compasses to any convenient Distance & place
one foot in the assigned point C & with the other make the Marks
C & D equidistant on each Side the point C. Then opening your
Compasses to any convenient Distance wider than the for-

mer place one foot in 6 & with the other describe the Arch F F. Then place one foot in D & with the other describe the Arch G g. Then from the point where these two Arches intersect or cut each other draw a right Line to the assigned point & which is the thing required.

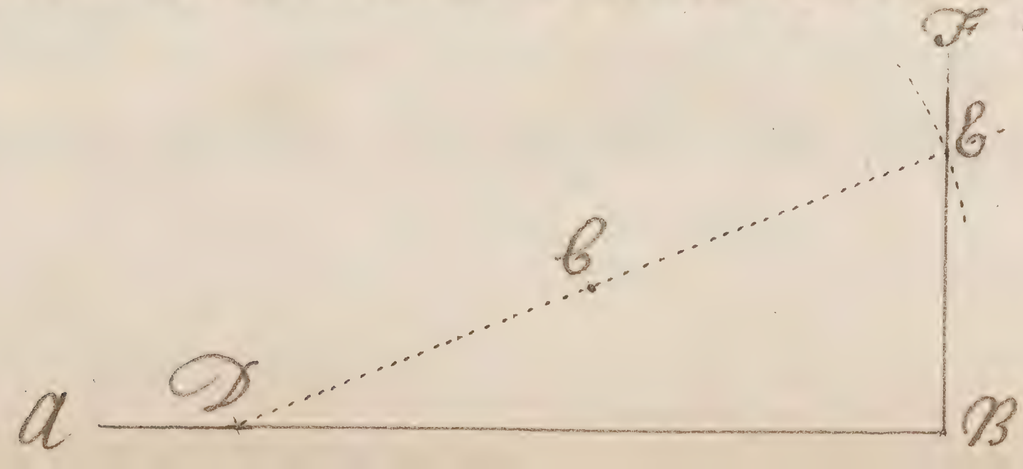


Problem 2.

How to erect a perpendicular upon the End of a right Line given.

Let the right Line A B be given & from the End Thereof at B let it be required to erect the perpendicular B F.

Open your Compasses to any convenient Distance & place one foot in B & with the other make the mark C where you must rest the compasses point & with the other make the Mark D in the given Line A B & then lay a Ruler from D to C & draw the line D E making C E = C D Lastly from B through E draw the Line B F which is the perpendicular to the line given.

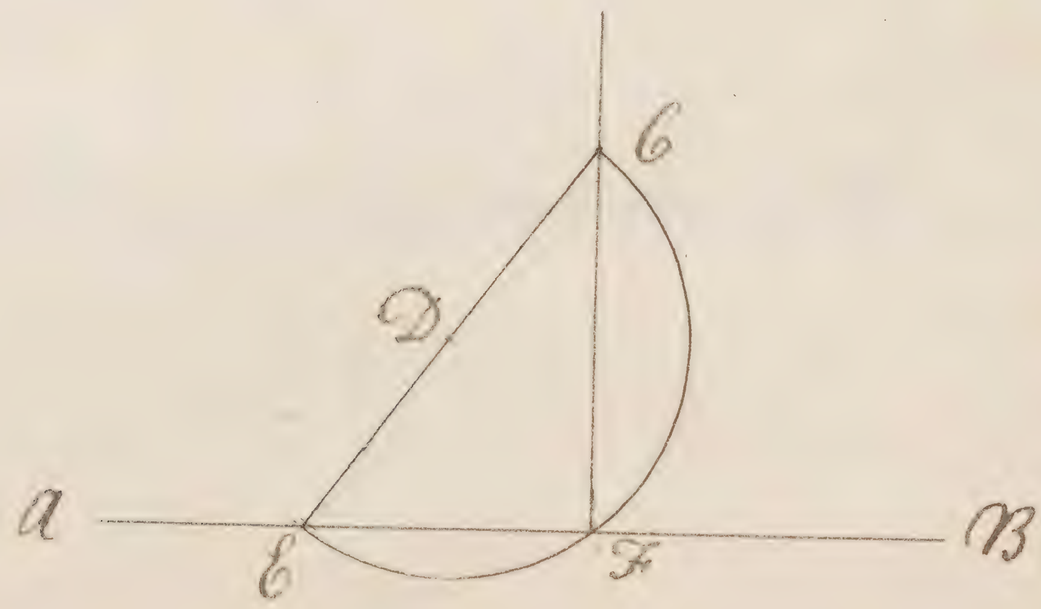


Problem 3.

How to let fall a perpendicular from any point assigned upon a right Line given.

The point given is C from which point it is required to draw a right Line which shall be perpendicular to the given right Line A B.

First from the point C given draw a line by Chance to the line A B as C C which divide into two equal parts in the point D then placing one foot of the Compasses on the Point D with the Distance D C describe the Semicircle C F C cutting the given Line A B in the point F. Lastly if from the point C you draw the right Line C F it shall be perpendicular to the given line A B which was requir.



Problem 4.

How to make an Angle equal to an Angle given.

The Angle given is A C B & it is required to make another Angle equal thereunto.

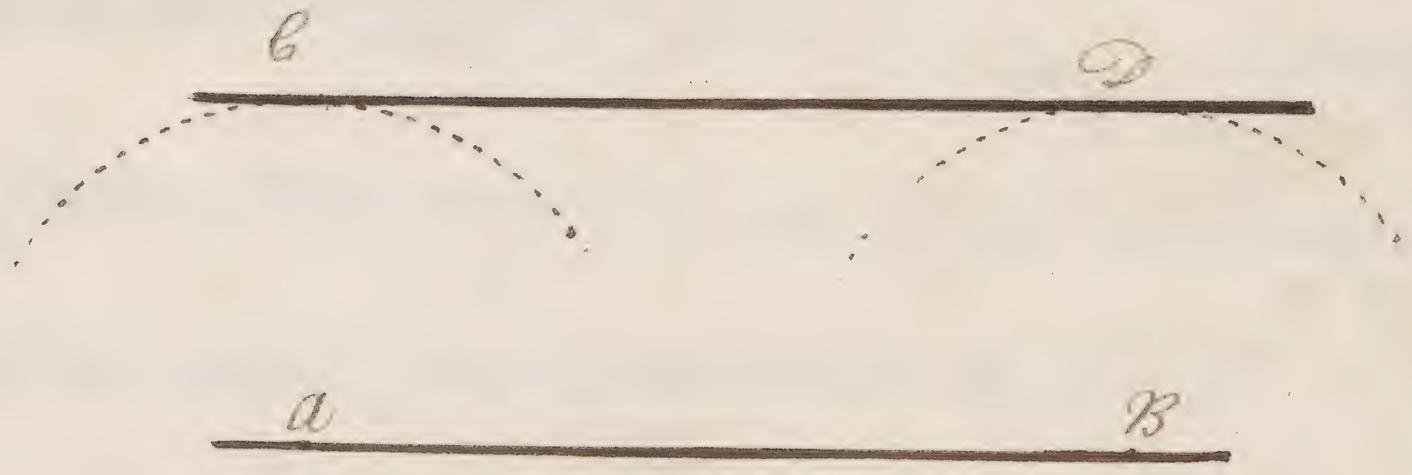
First draw the Line E F at pleasure then upon the given Angle at C the compasses opened at any distance describe the Arch A B also upon the Point F (the Compasses unaltered) describe the Arch D E then take with your Compasses the Distance A B & set the same Distance from E to D. Lastly draw the Line D F so shall the Angle D F E be equal to the Angle A C B.

Problem 5.

A right Line being given how to draw another right Line which shall be parallel to the former at any Distance.

The given Line is A B unto which it is required to draw another right Line parallel thereunto at the Distance M N.

First open your Compasses to the Distance M N then placing one foot in A with the other describe the Arch b also place one foot in B with the other describe the Arch D. Lastly draw the Line b D so that it may only touch the Arches b & D so shall the Line b D be parallel to the Line A B at the Distance M N as was required.

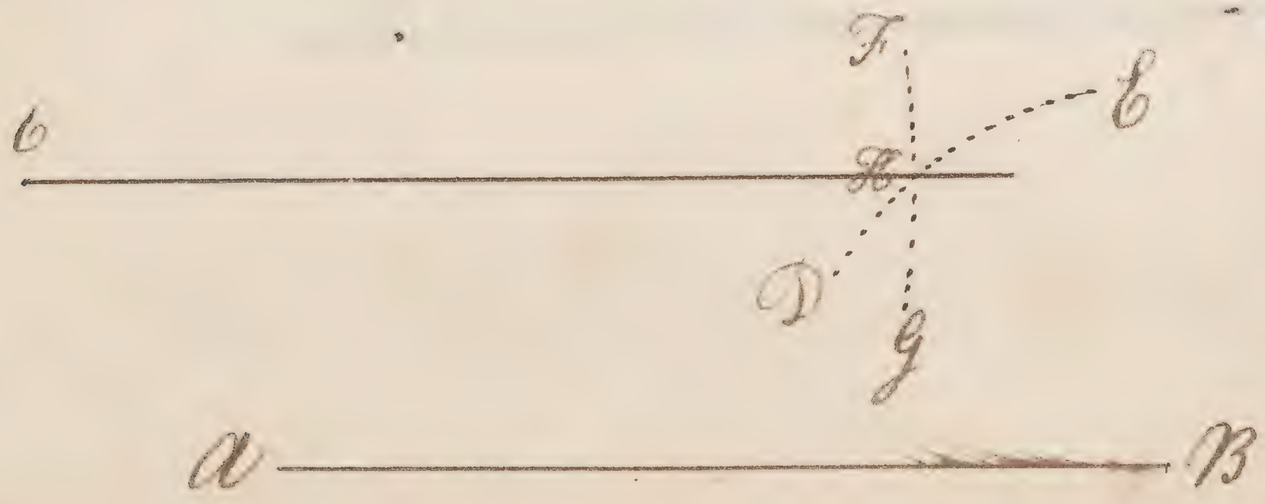


Problem 6.

A Right Line being given how to draw another right Line paral-lel to the same which shall also pass through a Point assigned.

Let A B be the Right Line given to which let it be required to draw a parallel line from a Point not over the given Line.

First take with your Compasses the Distance from A to C & placing one foot whereof in B with the other describe the Arch D E then take in your Compasses the whole Line A B & placing one foot in the point C & with the other describe the Arch F G cross-ing the for-mer Arch D E in the point H. Lastly if you draw the Line C H it shall be parallel to the Line A B.

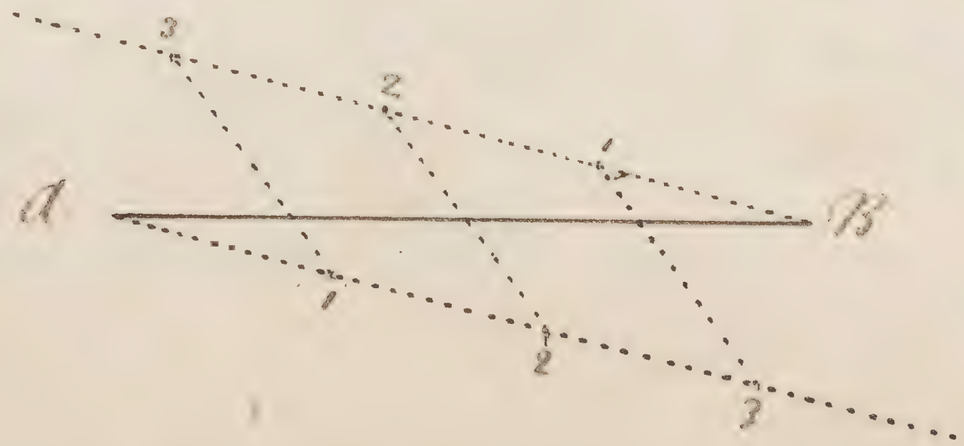


Problem 7.

To divide a right line into any number of equal parts.

Let it be required to divide the given right Line A B into four equal parts

First from the End of the given Line A draw the Line A C making an acute Angle then from the other End of the given Line at the point B draw the line B D parallel to A C (or make the A B D equal to the Angle C A B) then upon the lines A C & B D set off any three equal parts (which must be always one less than the Number of parts the given line is to be divided into) on each line as 1, 2, 3 then draw lines from 1 to 3 from 2 to 2 from 3 to 1 which lines crossing the given line A B shall divide it into four equal parts as was required.

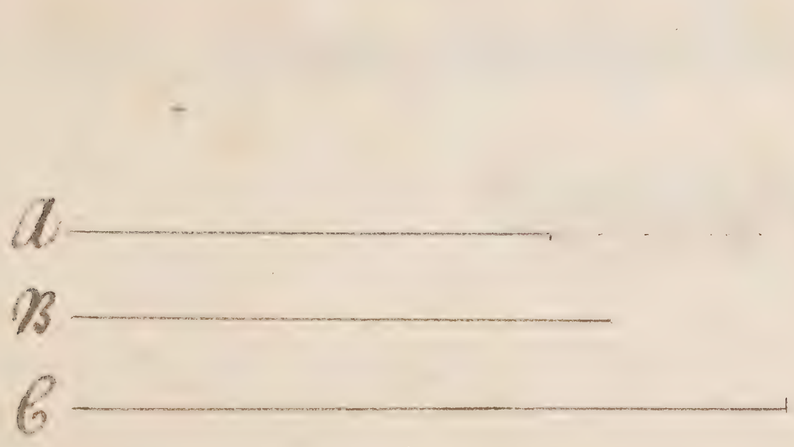


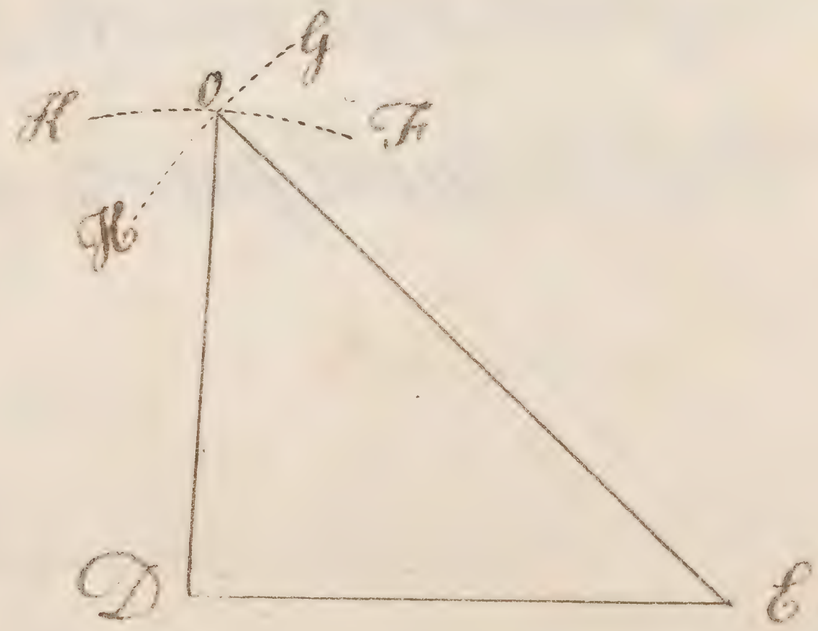
Problem 8.

Any three right Lines being given so that the two shortest toge-ther be longer than the the third to make thereof a Triangle.

Let it be required to make a Triangle of the three Lines A B & C the two shortest whereof Viz. A & B together are longer than the third line C.

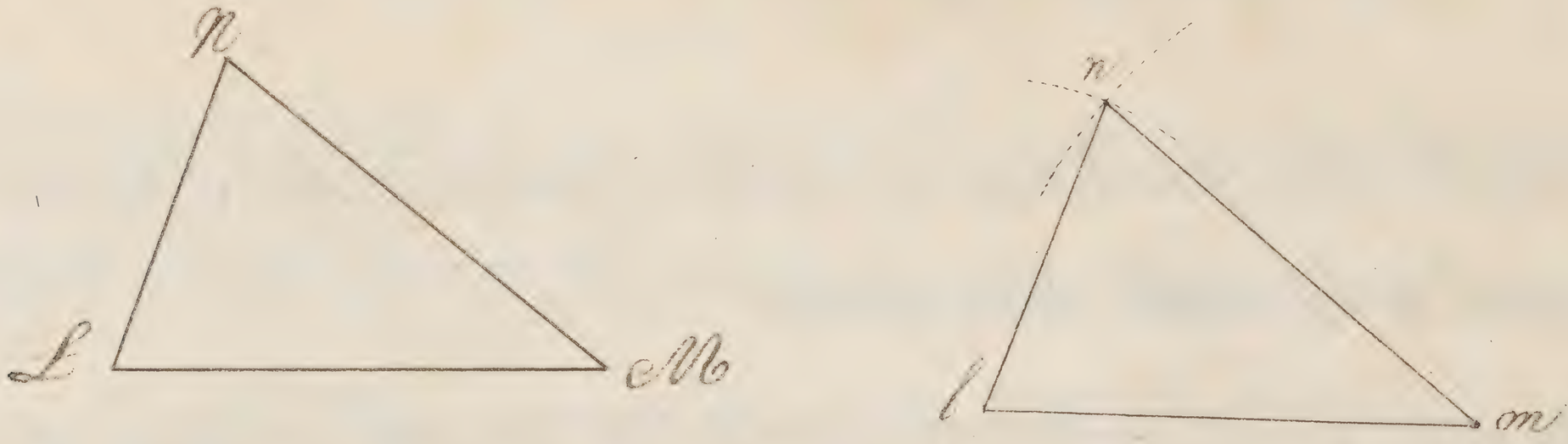
First draw the Line D E equal to the given line B then take with your compasses the Line C & setting one foot in E with the other describe the Arch H G: also take the given line A in your compasses & placing one foot in D with the other describe the Arch K F cutting the former Arch H G in the point O. Lastly if you draw from the point O the lines O E & O D you shall constitute the Triangle O D E whose sides shall be equal to the three given Lines A, B & C.



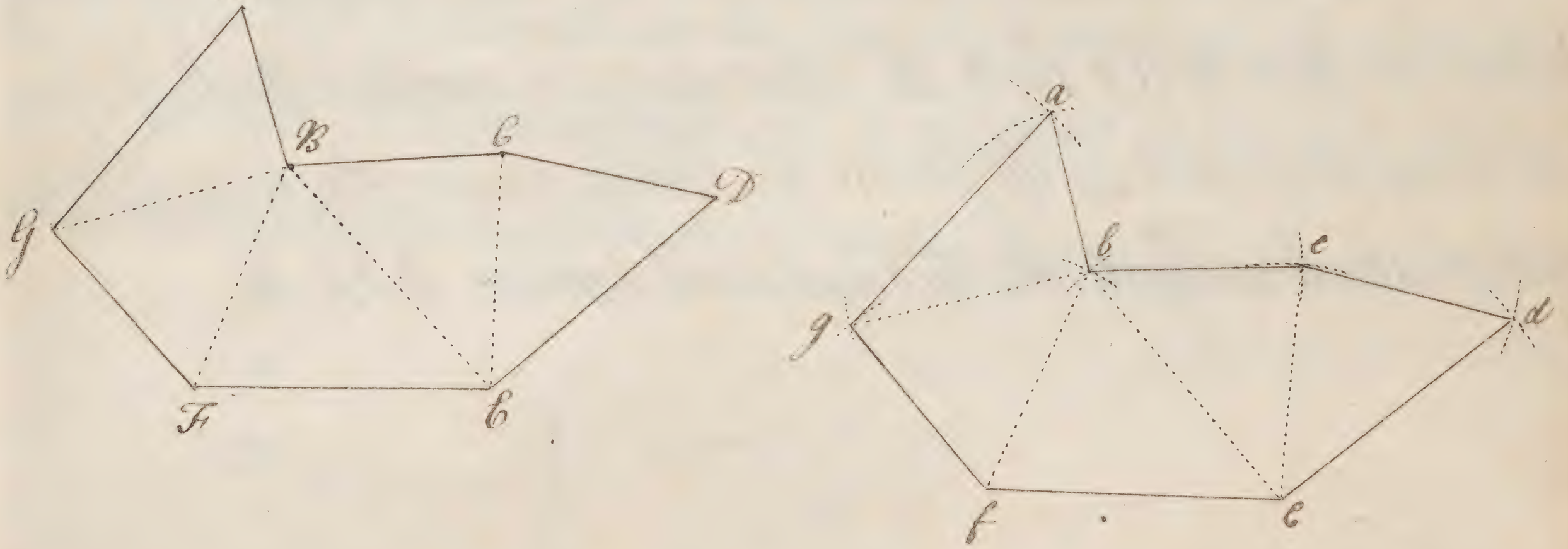


In like manner a Triangle l m n may be made equal to

Lelle N another given.



Also any rectilineal Figure a b c d e f g equal to any other rectilineal Fi-
gure A B C D E F G by making the Triangles a b g, b g f, b f e, b c e, c e d each respec-
tively equal to the triangles A B G, B G F, B F E, B C E, C E D.

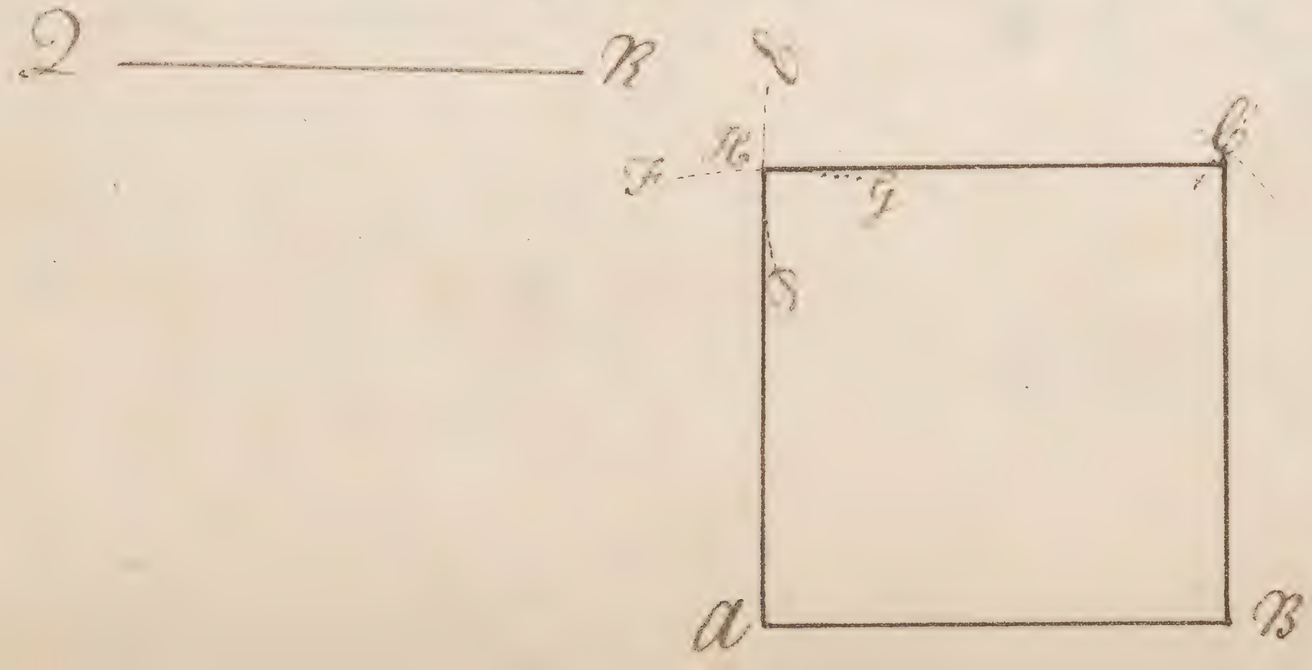


Problem 9.

Having a Right line given, how to make a Geometrical Square whose side shall be equal to the right Line given.

The line given is 2 K & it is required to make a Geometrical Square whose side shall be equal to the line 2 K.

First draw the line A B making it equal to the given line 2 K, then (by the first or second Problem) upon the point B erect the perpendicular B C making the line B C equal to the Given line 2 K also; then taking the line 2 K in your compasses place one foot in C & with the other describe the Arch D C; also the compasses so resting place one foot in A & with the other describe another Arch F G crossing the former in the point E. Lastly draw the lines E C & E D which shall include the Geometrical Square A B C E.



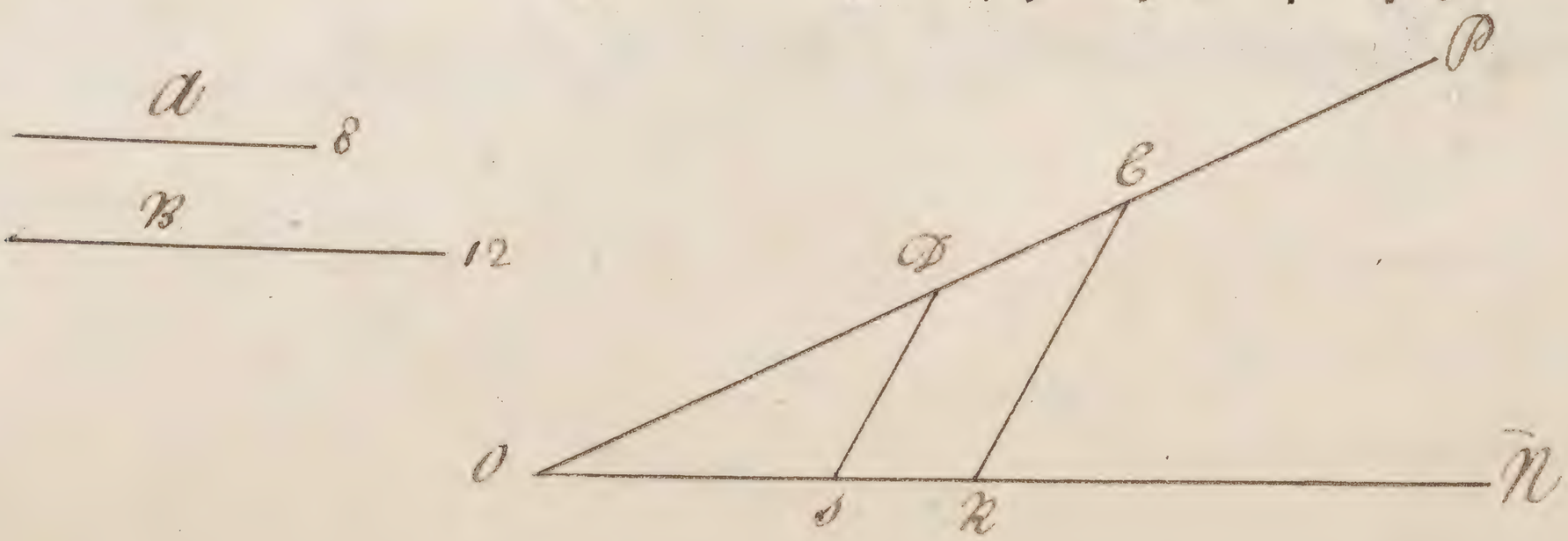
Problem. 10.

Two right lines being given, how to find a third right Line which shall be in proportion unto them.

Let the two given lines be A & B & let it be required to find a third line which shall be in proportion unto them.

First draw two right Lines making any angle at pleasure as the Lines O P & O N making the angle P O N; then taking the line A in your Compasses set the length thereof from O to S; Also take the (line) *B in your Compasses & set the length thereof from O to R; & also from O to D: then draw the line S D & from the point R draw the right line R C parallel to S D: so shall O C be the third proportional required. For, As O S is to O D so is O R to O C.*

8 : 12 :: 12 : 18



Problem 11.

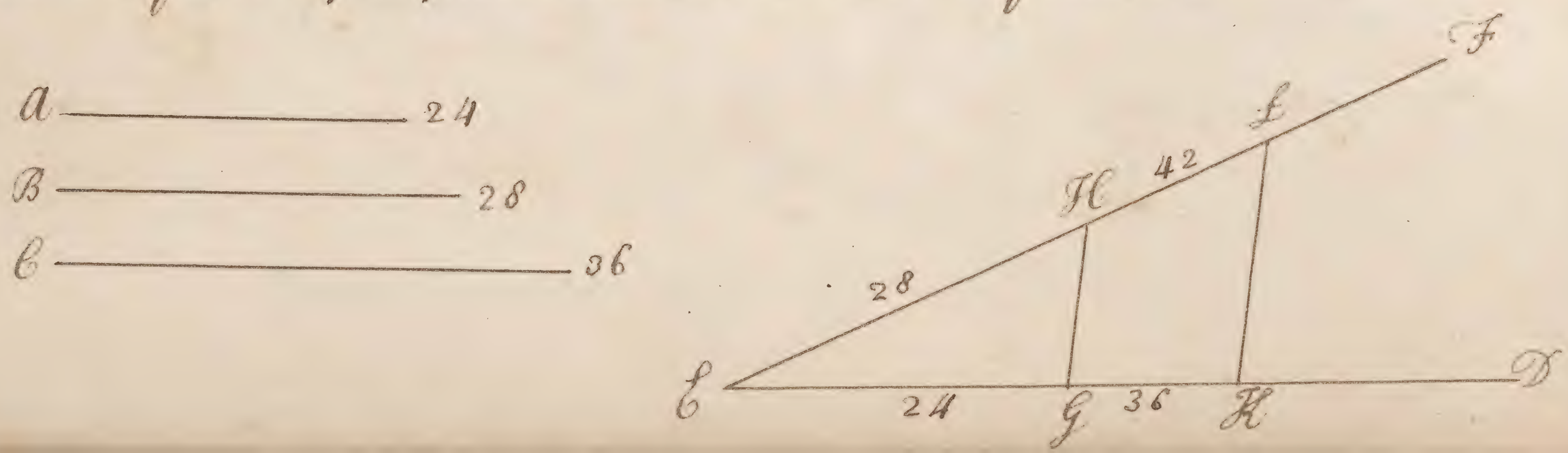
Three right Lines being given, to find a fourth in Proportion to them.

The three lines given are A, B, & C unto which it is required to find a fourth proportional line. This is to perform the Golden Rule, or Rule of 3 by lines.

As in the last Problem, you must draw two lines making any Angle as the Angle D & F; then take the line A in your Compasses & set it from & to G; then take the line B in your Compasses & set that length from & to H; then take the third given line in your Compasses & set that from & to K & through that Point K draw the line K L parallel to G H so shall the line & L be the fourth Proportional required. For As & G is to & H so is & K to & L

24 : 28 :: 36 : 42

N B. That in the performance of this Problem the first & third terms (viz the lines A & C) must set upon the same line as here upon the line & D, & the second Term (viz: the line B) must be set upon the other line & F upon which the fourth Proportional & L will be found.



Problem 12.

To divide a right line given into two parts which shall have such proportion one to the other as two given Right Lines.

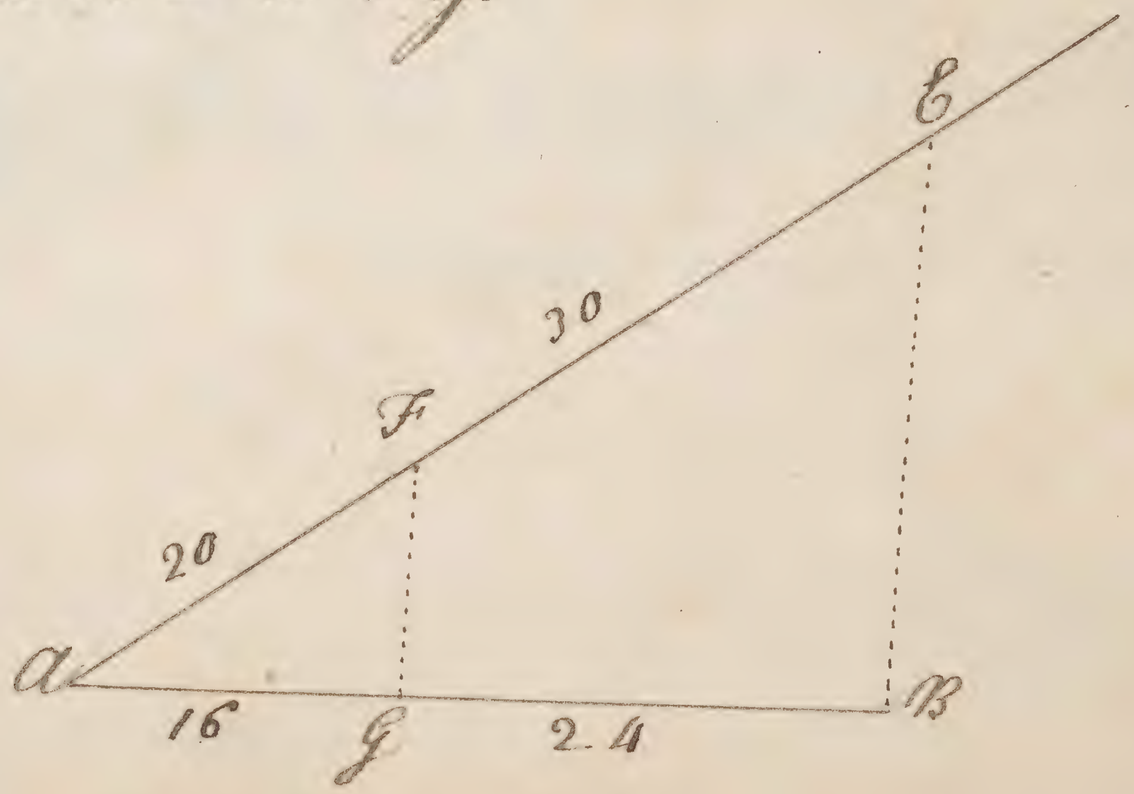
The line given is A B & it is required to divide the same into two parts, which shall have such proportion one to the other as the line C has to the line D.

First from the point A draw the line A E at pleasure making the Angle E A B, then take in your compasses the line C & set it from A to F, also take the line D & set it from F to E & draw the line E B, then from the point F draw the line F G parallel to E B cutting the given line A B in the point G, so is the line A B divided into two parts in the point G, being in proportion one to the other as the line C is to the line D: For

As A E is to A B so is A F to A G.

C ——— 20

D ——— 30



Arithmetically.

Let the Line AB contain 40 Perches, & let the line C be 20, & the line D 30; & let it be required to divide the line AB into two Parts being in proportion one to the other as the line C is to the line D.

First add the lines C & D together their Sum is 50; then say by the Rule of Proportion If 50 (which is the Sum of the two given terms) give 40, the whole line AB, what shall 30, the greater given Term, give? Multiply & divide & you shall have in the Quotient 24 for the greater part of the line AB which being taken from 40 the whole line there remains 16 for the other Part AG. For

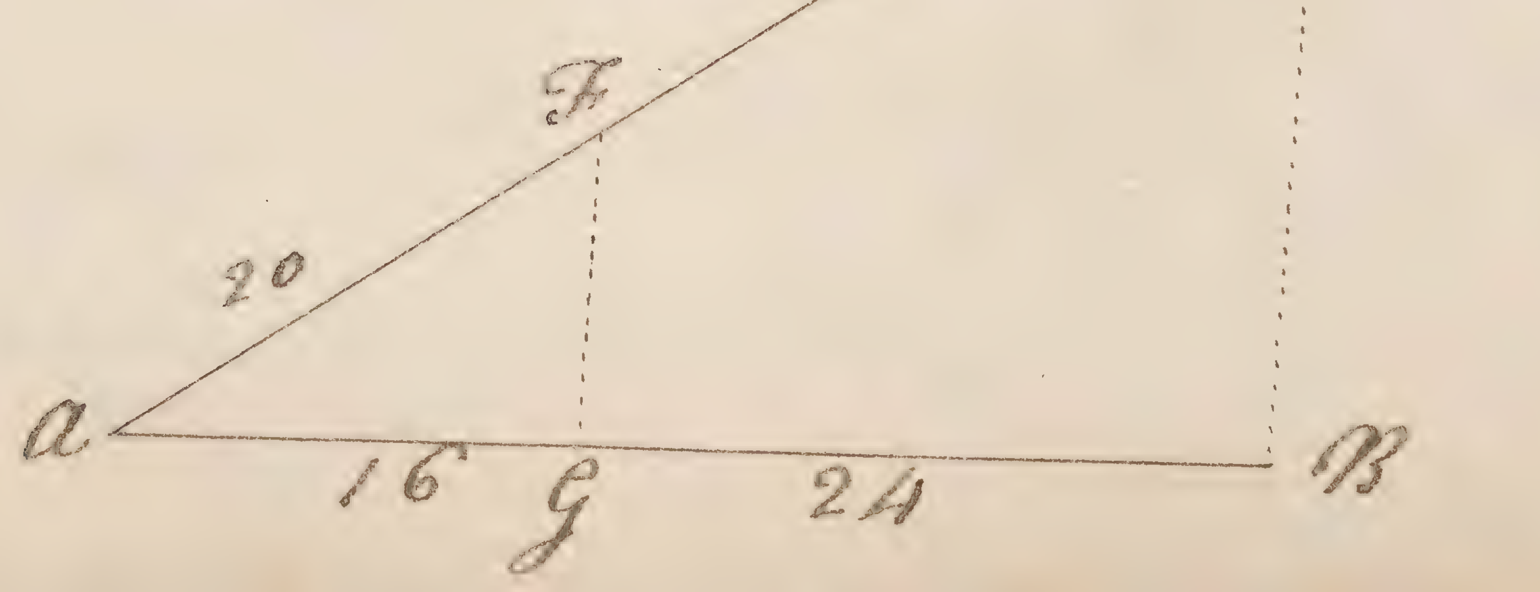
As AE is to AB so is FE to GB.

50 : 40 :: 30 : 24

30

5/0 | 120|0 | 24

40
24
16



Problem 13.

How to divide a Triangle into two parts according to any proportion assigned by a line drawn from any Angle thereof & to lay the lesser part towards any Side assigned.

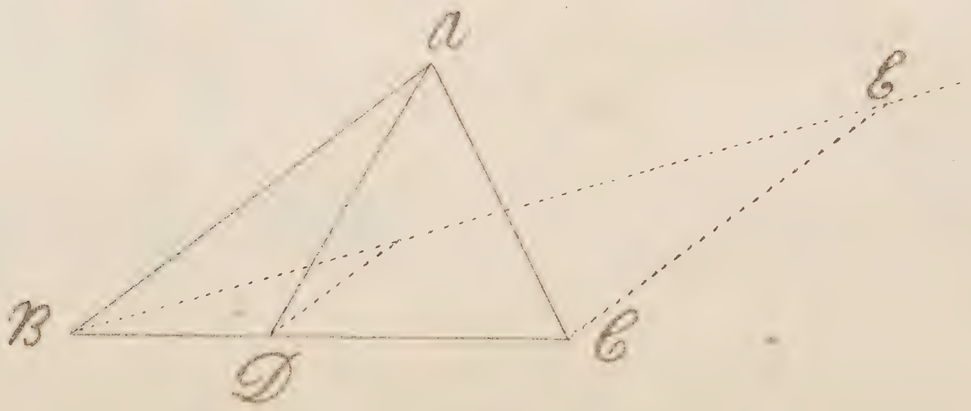
Let ABC be a Triangle given & let it be required to divide the same by a line drawn from the Angle A into two parts the one bearing proportion to the other as the line F does to the line G & that the lesser part may be towards the Side AB

By the last Problem divide the Base of the Triangle BC in the point D in proportion as the line F is to the line G the lesser part being set from B to D. Lastly draw the line AD which shall divide the Triangle ABC in proportion as F to G. For

As the line F is to the line G
So is the Triangle ADC to the Triangle ABD.

F ———————

G ———————



Problem 14.

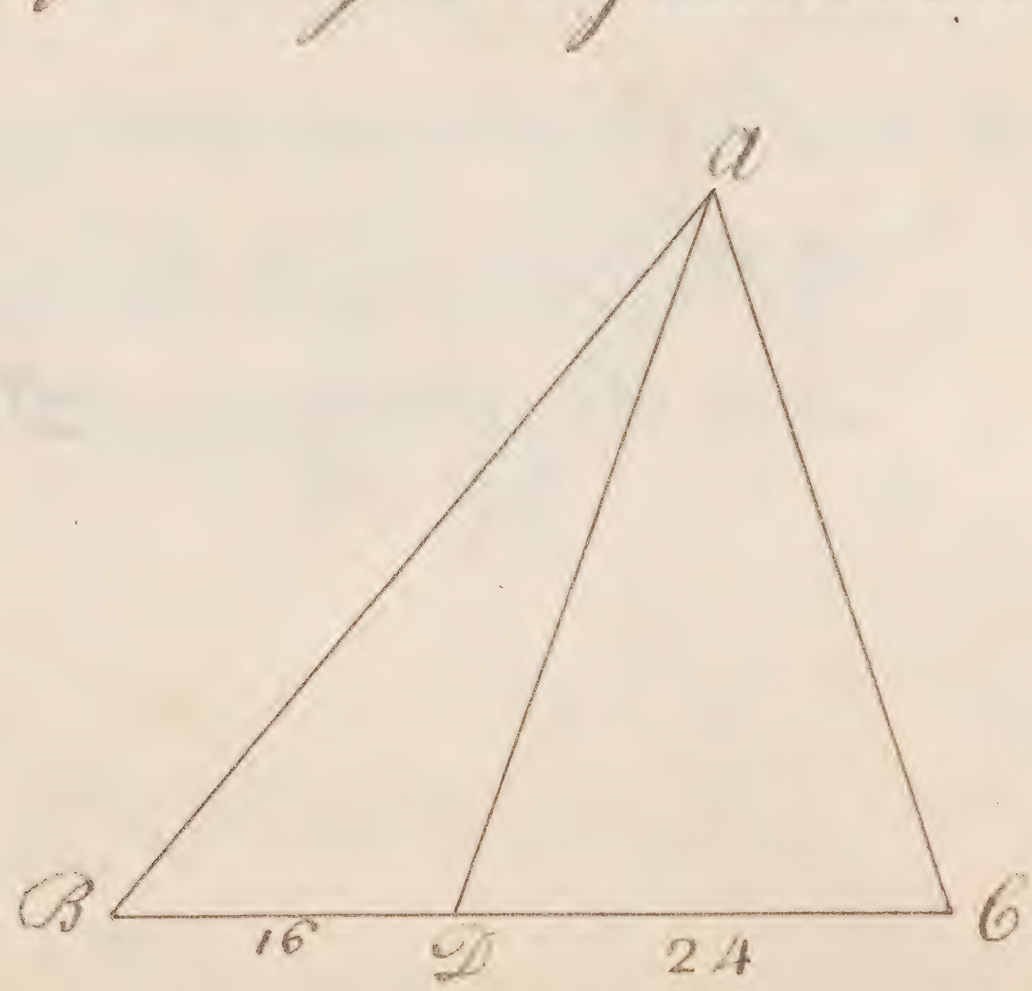
The Base ~~being~~ of the Triangle being given to perform the foregoing Problem by Arithmetick.

Suppose the Base of the Triangle BC to be 40 & let the Proportion into which the Triangle A BC is to be divided be as 2 to 3:

First add the two proportional Terms together 2 & 3 which makes 5; then say by the Rule of Proportion, if 5 (the Sum of the proportional Terms) gives 40 (the whole Base BC) what shall 3 (the greater Term) give? Multiply & Divide & the Quotient will give you 24 for the greater Segment of the Base DC, which being deducted from the whole Base 40 there will remain 16 for the lesser Segment BD.

5 : 40 :: 3

$$\begin{array}{r} 3 \\ 5\overline{)120} \\ 24 \end{array} \qquad \begin{array}{r} 40 \\ 24 \\ \hline 16 \end{array}$$



Problem. 15.

How to divide a Triangle whose Area or Content is known, into two parts by a line drawn from an Angle assign'd according to any Proportion required.

Let the Triangle ABC contain 8 Acres & let it be required to divide the same into two parts, by a line drawn from the Angle A the one to ~~to~~ contain 5 Acres & the other 3 Acres.

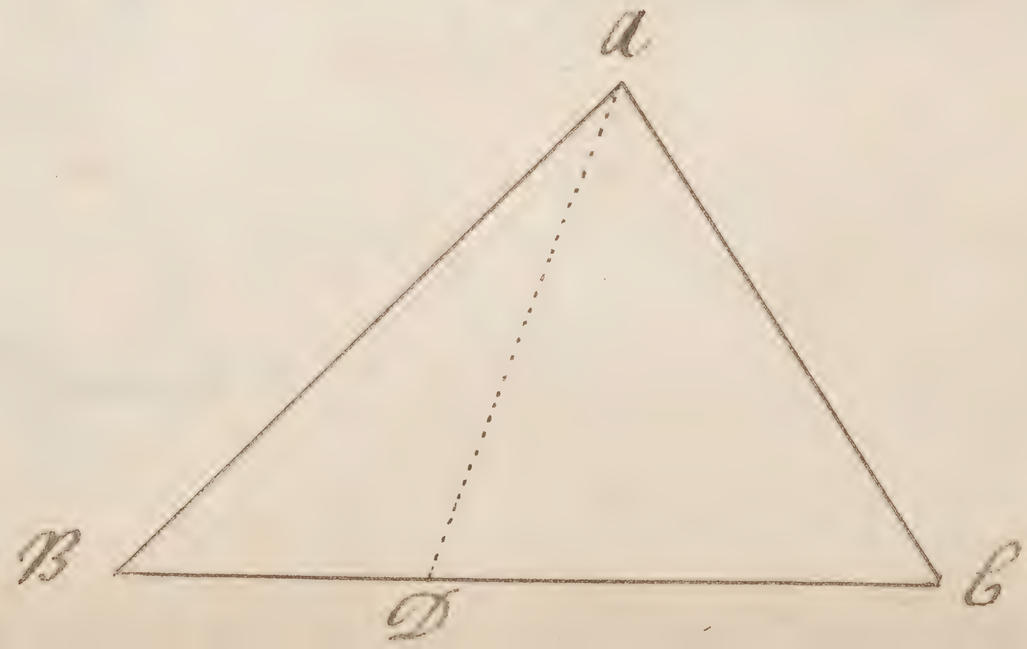
First measure the whole length of the Base which suppose 40, then say if 8 Acres (the Quantity of the whole Triangle) give 40 (the whole Base) what part of the Base shall 5 Acres give? Multiply & divide, & the Quotient will be 25 for the greater Segment of the Base CD, which being deducted from 40 (the whole Base) there will remains for the lesser segment of the Base BD; then draw the line AD which shall divide the Triangle ABC according to the Proportion required.

8 : 40 :: 5

5

8 | 200

25



Problem. 16.

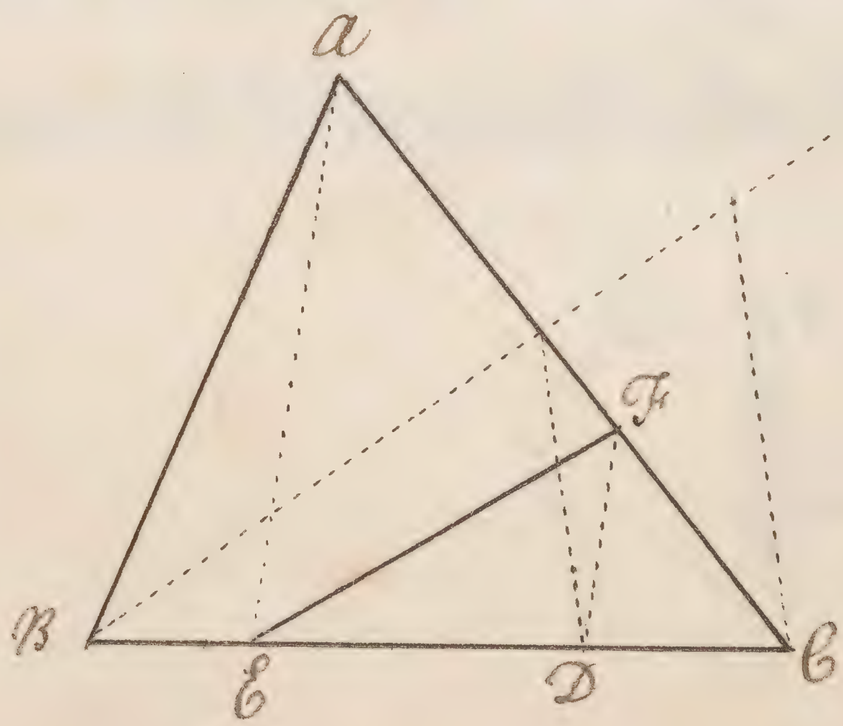
How to divide a Triangle given into two parts according to any proportion assigned by a line drawn from a point limited in any of the Sides thereof; & to lay the greater or lesser part towards an Angle assigned.

The Triangle given is ABC & it is required from the point E to draw a line that shall divide the Triangle into two parts being in proportion one to the other as the line I is to the line K & to lay the lesser part towards B.

First from the limited point E draw a line to the opposite Angle at A; then divide the Base BC in proportion as I to K which point of division will be at D then draw D F parallel to A E. Lastly from F draw the line F E, which will divide the Triangle into two parts being in proportion one to the other as the line I is to the line K.

I ——

K ——————



Problem. 17.

To perform the foregoing Problem Arithmetically.

It is required to divide the Triangle ABC from the point C into two parts in proportion as 5 to 2.

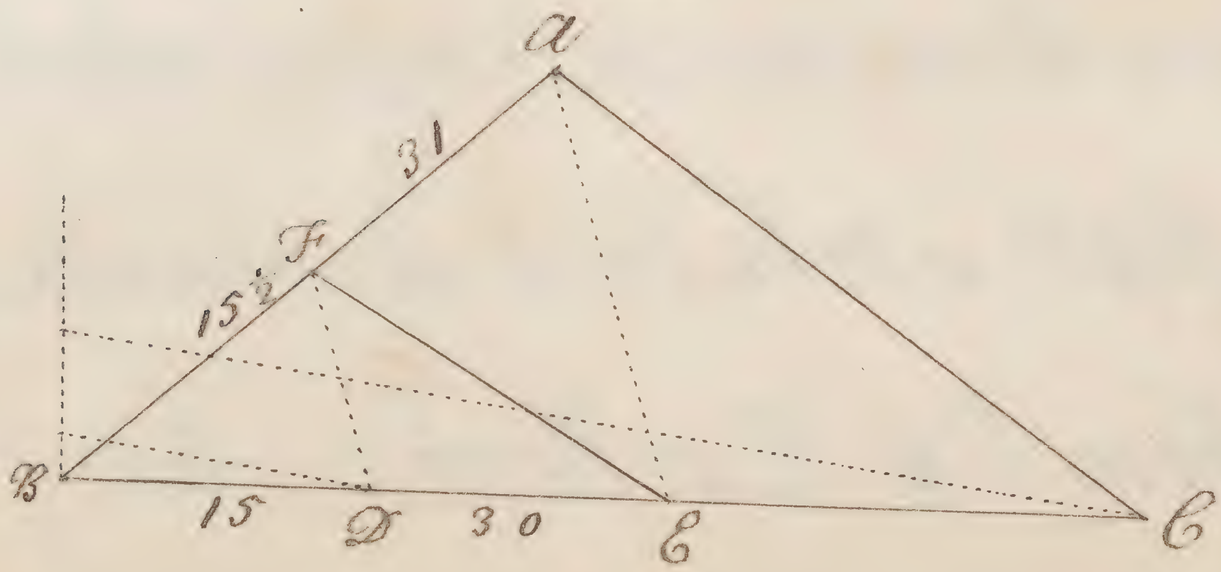
First divide the Base BC according to the given proportion, then (because the lesser part is to be laid towards B) measure the Distance from C to B which admit 30 then say by the Rule of Proportion If C B 30 give D B 15 what shall A B 31 give? Multiply & divide & the Quotient will be 15½ which lay from B to F then from F draw the line F C which shall divide the Triangle according to the required proportion.

30 : 15 : : 31

15

30) 465

15.5



Problem 18.

How to divide a Triangle whose Area or Content is known into two parts by a line drawn from a point limited in any Side thereof according to any Number of Acres Roods & Perches.

In the foregoing Triangle ABC whose Area or Content is 5 Acres 1 Rood let the limited Point be C in the Base thereof & let it be required from the Point C to draw a right Line which shall divide the Triangle into two parts between M & N so that M may have 3 Acres 3 Roods thereof & N may have 1 Acre & 2 Roods thereof.

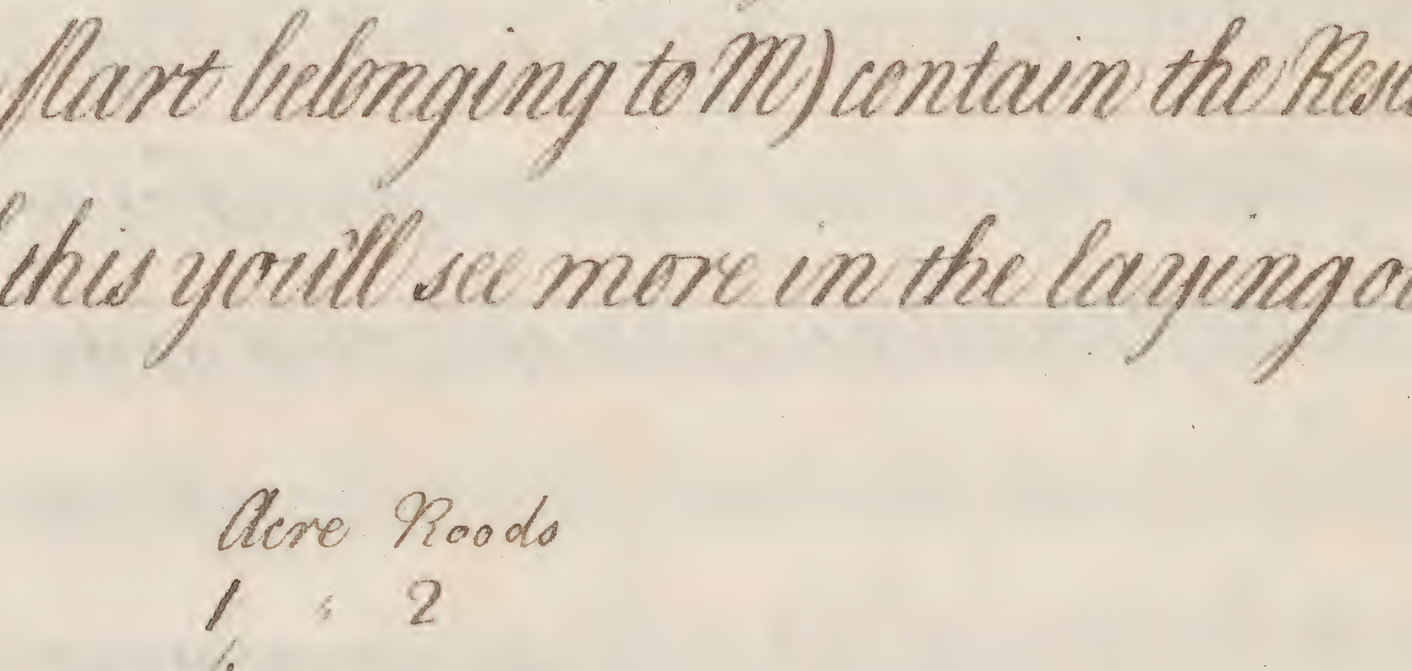
First reduce the Quantity of N (being the lesser) into Perches which make 240 then (considering on which Side of the limited Point C this part is to be laid as towards B) measure that part of the Base from C to B 30 Perches whereof take the half which

is 15 & thereby divide 240 the part belonging to N the Quotient will be 16 the length of the perpendicular F H, at which parallel Distance from the Base B C cut the side A B in F from whence draw the line F E which shall cut off the Triangle F B E, containing 1 Acre 2 Roods the part belonging to N, then will the Trapezia A F E C (which is the part belonging to M) contain the Residue Viz 3 Acres 3 Roods.

But of this you'll see more in the laying out of Lands.

Acre	Roods
1	2
4	
6	
40	

$15 \left\{ \begin{array}{l|l} 3 & 240 \\ \hline 5 & 80 \\ \hline & 16 \end{array} \right.$

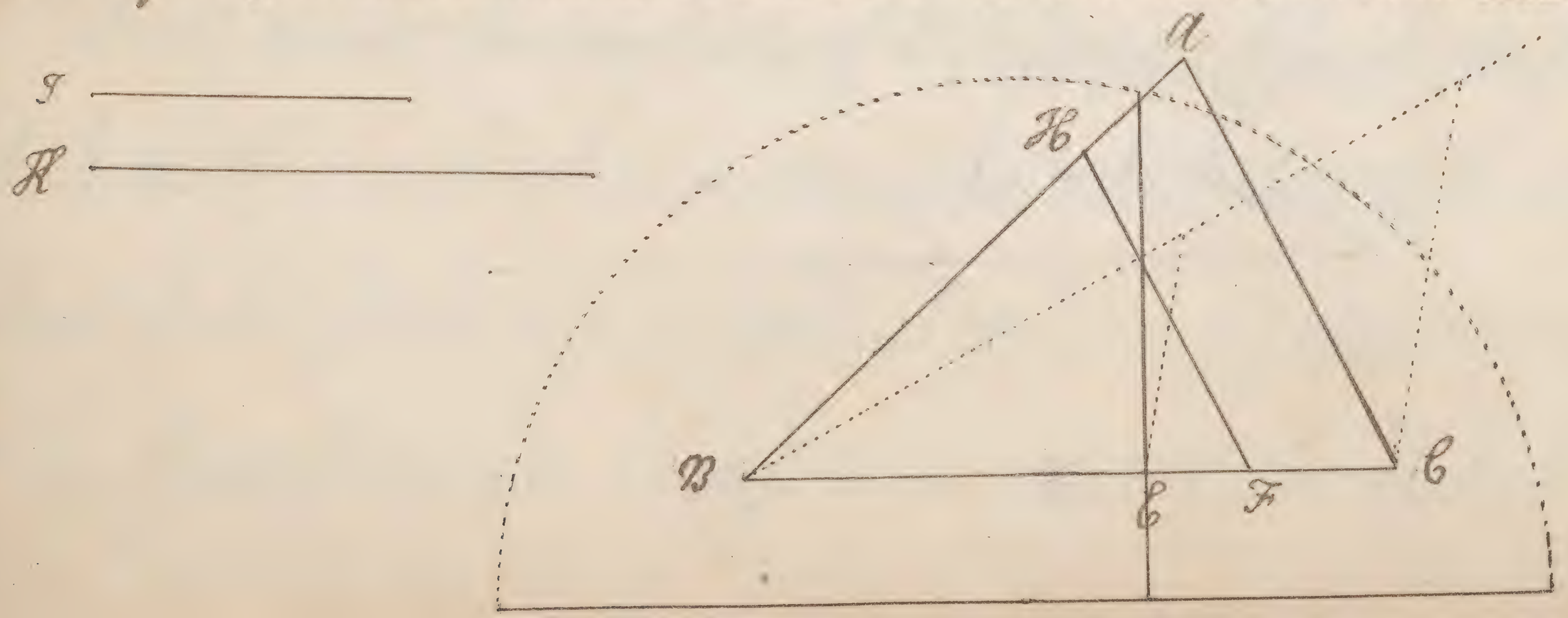


Problem 19.

How to divide a Triangle according to any Proportion given by a line drawn parallel to one of the Sides given.

The Triangle ABC (fig: 29) is given & it is Required to divide the same into two parts by a line drawn parallel to the Side AC which parts shall be in proportion one to the other as the line I - to the line K.

First (by Prob: 12) divide the line BC in E in proportion as I to K; then (by Prob: 23. following) find a Mean proportional between BE & BC which let be BF from which point F draw the line FH parallel to AC which line shall divide the Triangle into two parts Viz: the Trapezia AHFE & the Triangle HFB which are in proportion one to the other as the line K is to the line I.



Problem 20.

To perform the foregoing Problem Arithmetically.

Let the Triangle be ABC (fig: 29) & let it be required to divide the same into two parts which shall be in proportion one to the other as 4 to 5 by a line drawn parallel to one of its Sides.

First let the Base BC containing 54 be divided according to the proportion given so shall the lesser Segment BC contain 24 & the greater Segment CC 30: then find out a Mean proportional line between BC 24 & the whole Base BC 54, by multiplying 54 into 24 whose Products is 1296 the Square Root whereof is 36 the mean proportional sought which is BF: then through F draw a line parallel to AC & it is the Partition required; or else by the Rule of Proportion say If BC 54, give BF 36, what will AB 39 give? Answer BK 26: therefore draw KF & it is the Partition requir.

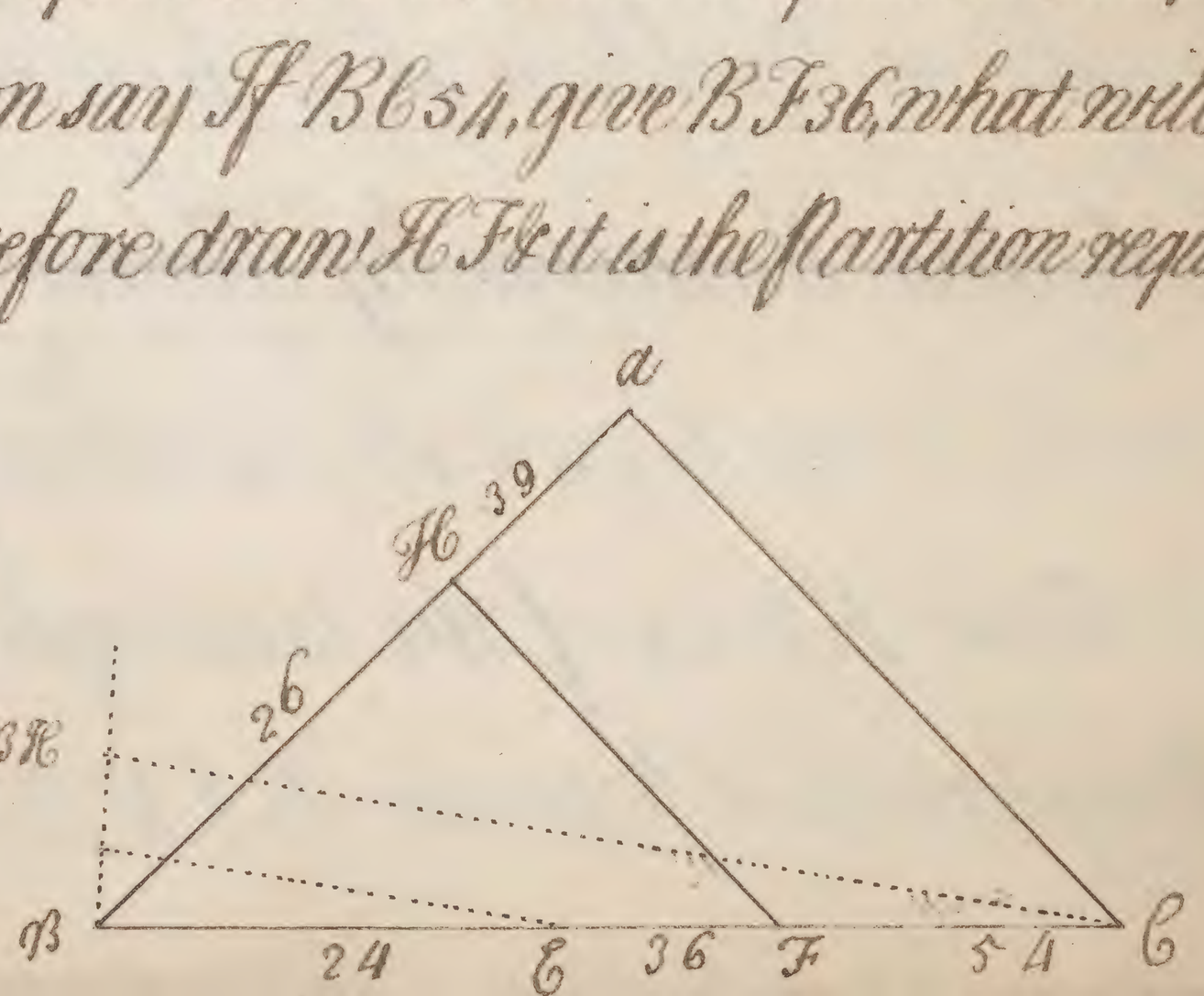
24 = BC
54 = BC

96
120

1296 (36 = BF
9
---
66) 396
396

BC BF AB
54 : 36 :: 39
36

234
117
---
54 { 6 | 1404
9 | 234
---
26 = BK



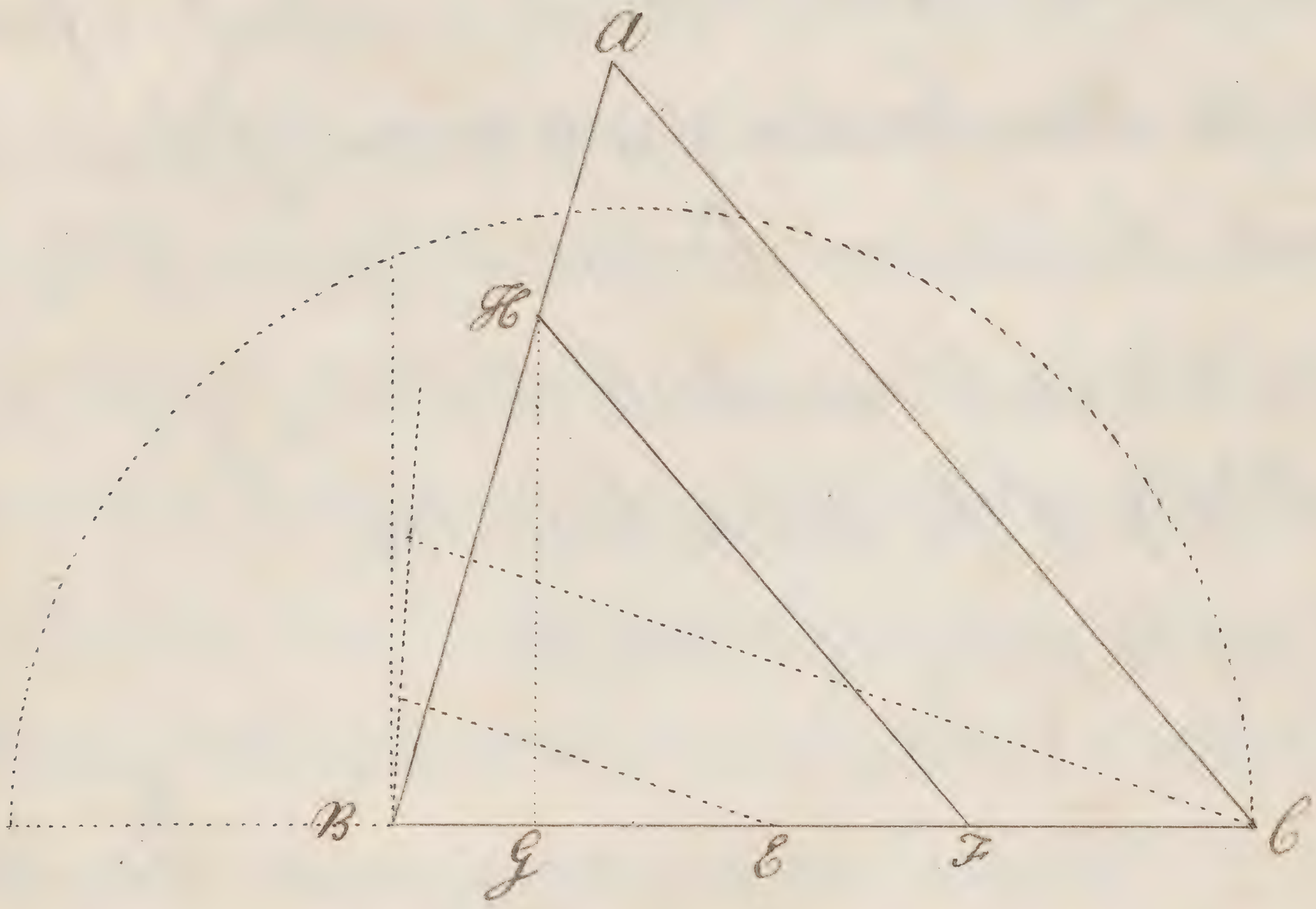
Problem 21.

To divide a Triangle of any known Quantity into two parts by a line drawn parallel to one of the Sides according to any Number of Acres Roods & Perches.

The Triangle given is ABC (fig: 29) whose Quantity is 8 Acres 0 Roods 16 Perches, & it is required to divide the same (by a line drawn parallel to the Side AC) into two parts Viz: 4 Acres 2 Roods 0 Perches & 3 Acres 2 Roods 16 Perches.

First reduce both Quantities into Perches (as is hereafter taught in Problem 28) & they will be 720 & 576; then reduce both these Numbers by Abbreviation into the least proportional Terms Viz 5 & 4 & according to that proportion divide the Base BC 54 of the triangle in E; then seek the Mean Proportional between BE & BC which proportional is

B F 36 of which 36 take the half & thereby divide 576 the
lesser Quantity of Perches the Quotient is H G 32 at which
parallel Distance from the Base cut off the line A B in H from
whence draw the line H F parallel to the side A C which
shall divide the triangle given according as was required.
But of this you'll see more in the laying out of Lands.



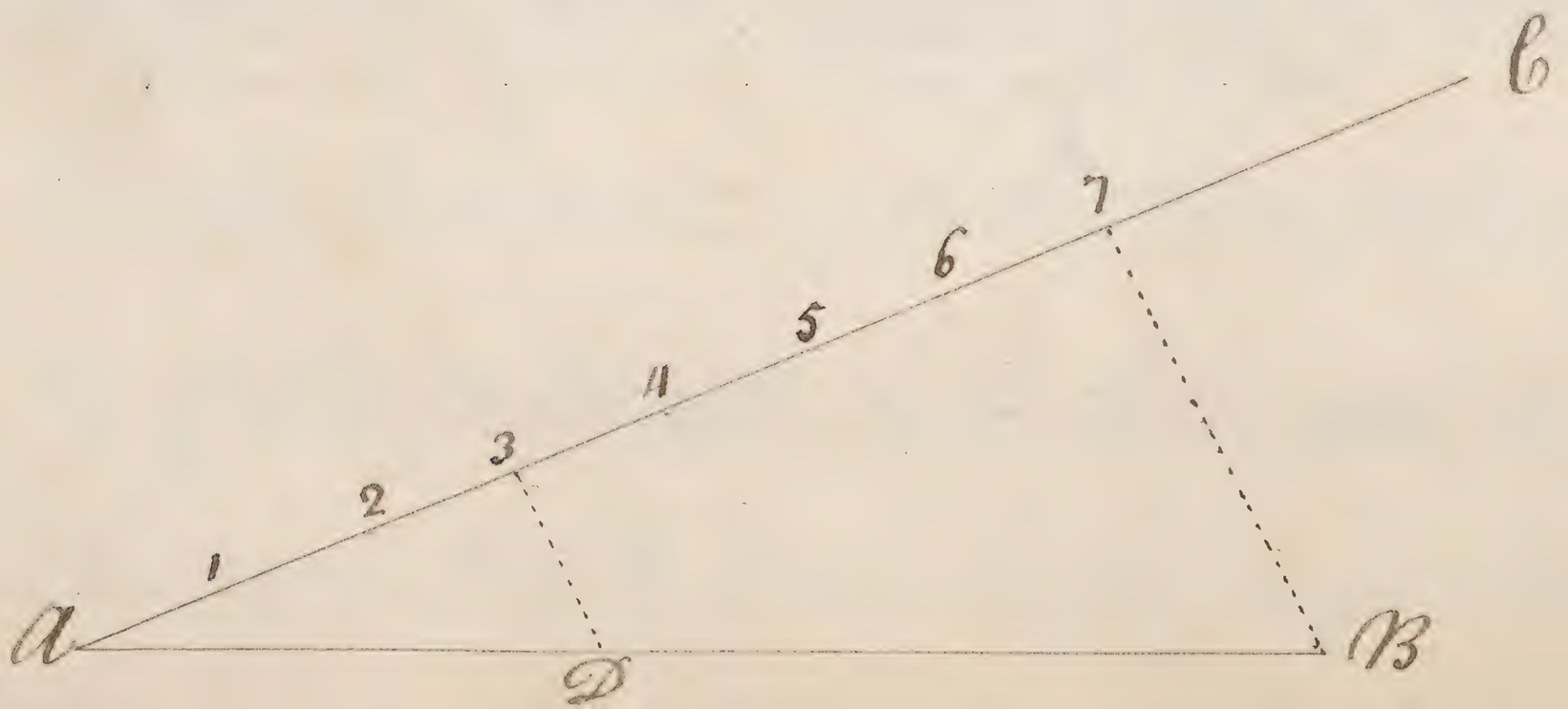
Problem 22.

From a line given to cut off any parts required.

The line given is AB (fig: 30) from which it is required to cut off $\frac{3}{7}$ parts.

First draw the line A C making any Angle as C A B. then from A set off any seven equal parts as 1. 2. 3. 4. 5. 6. 7 & from 7 draw the line 7 B. Now because $\frac{3}{7}$ is to be cut off from the line AB; therefore from the 3 draw the line 3 D parallel to 7 B cutting the line A B in D. so shall A D be $\frac{3}{7}$ of the line AB & D B shall be $\frac{4}{7}$ of the same line. For,

As A 7 is to A B so is A 3 to A D.



Problem 23.

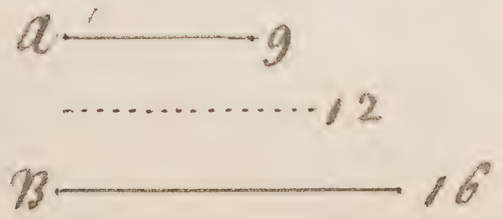
To find a Mean proportional between two lines given.

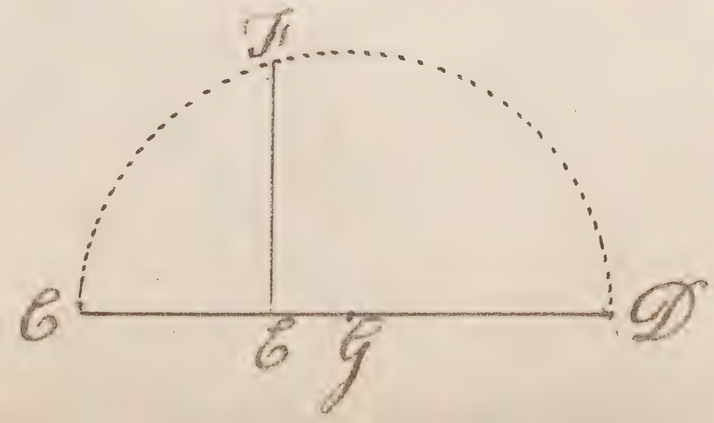
In fig: 31 let the two lines given be A 9 B 16 between which it is required to find a Mean proportional.

Let the two given lines A & B be joined together in the point E making one right line as C D which divide into two equal parts in the point G upon which point G with the Distance G C or G D describe the Semicircle C F D, then from the point E (where the two lines are joined together) raise the Perpendicular E F cutting the Periphery of the Semicircle in F so shall the line E F be a mean proportional between the two given lines A & B. for As C Dist to E F so is E F to E D

9 : 12 :: 12 : 16

In Numbers thus, multiply the Numbers expressing the given lines together, & extract the Square Root of the Product, which Root gives the Length of the mean proportional sought.





16
9
144 (12
1
22) 44
44

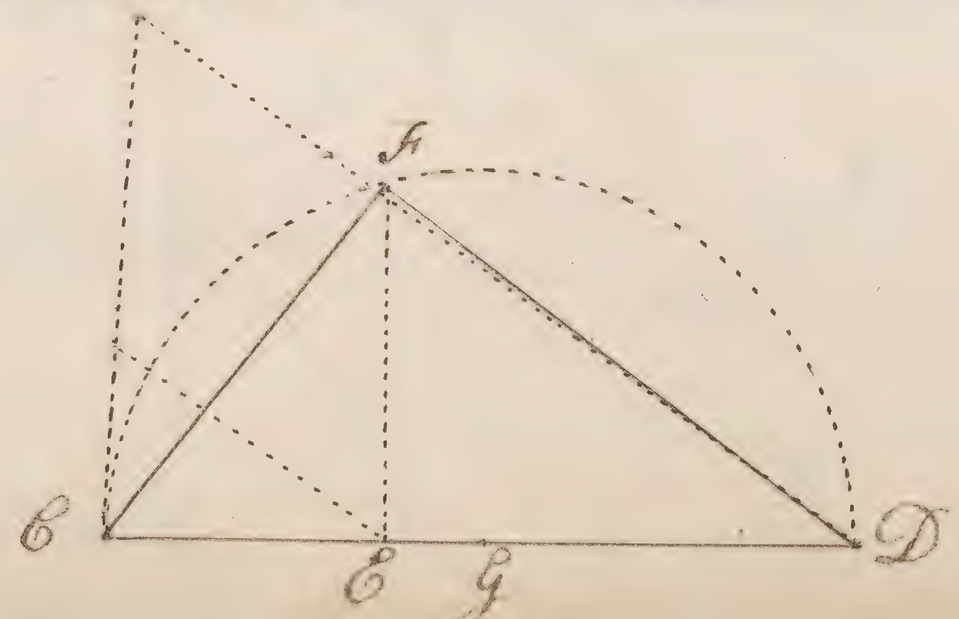
Problem 24.

How to find two lines which together shall be equal in power to any line given & in power the one to the other according to any proportion assigned.

In fig: 31 let C D be a line given whose power shall be equal to the sum of the powers of two sought lines which shall be to one another as a 9 to B 16.

First divide the line C D in the point G in proportion as a to B (by Prob: 12) then divide the line C D into two equal parts in the point G & upon G as a centre at the Distance G D describe the semicircle C F D & on the point G raise the perpendicular G F cutting the semicircle in F. Lastly draw the lines C F & D F which together in power shall be equal to the power of the given line C D & yet in power one to the other as A to B.

B ——— 16
a ——— 9

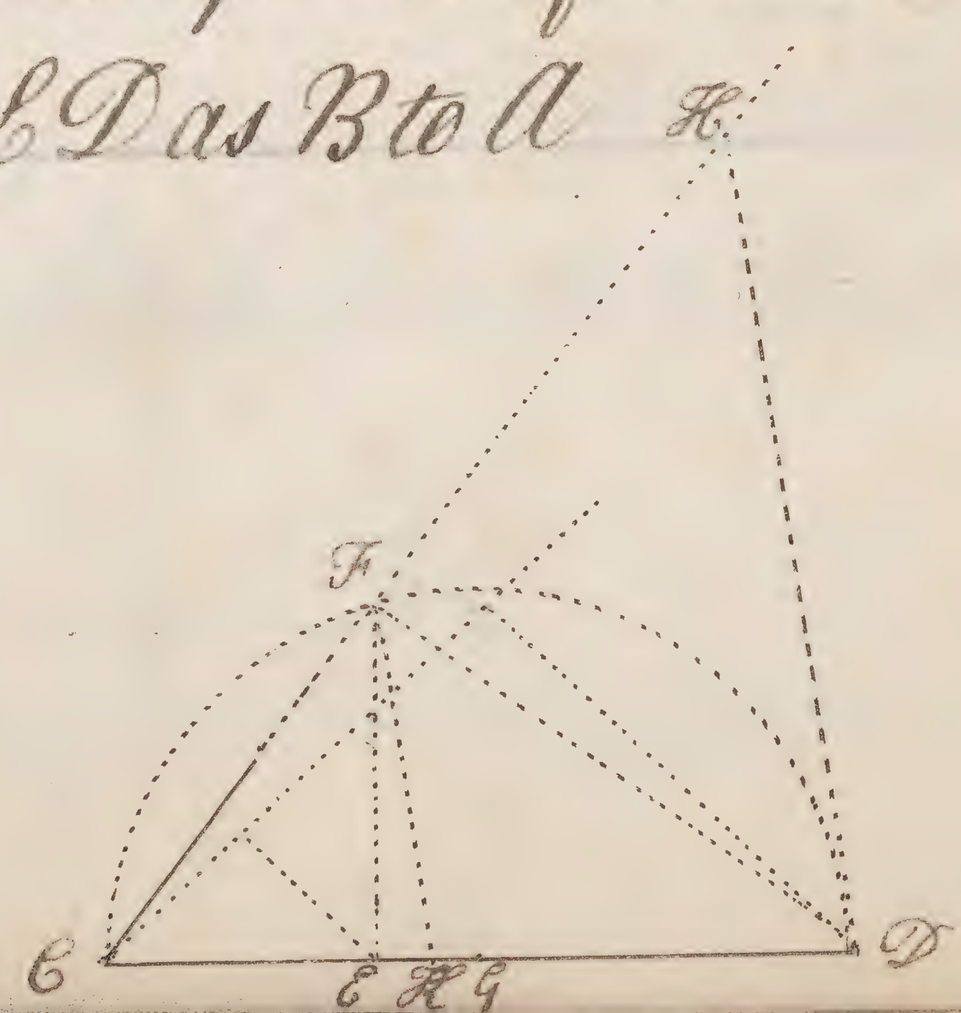


Problem 25.

How to divide a line in Power according to any Proportion given.

First divide the Line D 6 (fig: 32) in the Point E in proportion as A 9 to B 16 then divide the line 6 D into two equal parts in the Point G & upon G at the Distance G D describe the Semicircle 6 F D & on the Point E raise the Perpendicular E F cutting the Semicircle in F then draw the line 6 F & D F & produce the line 6 F to H till F H be equal to F D & draw the line H D & F K parallel thereunto; then shall the line 6 D be divided in K so that the Square of 6 K shall be to the Square of K D as 6 E to E D as B to A

B —— 16
A —— 9



Problem 26.

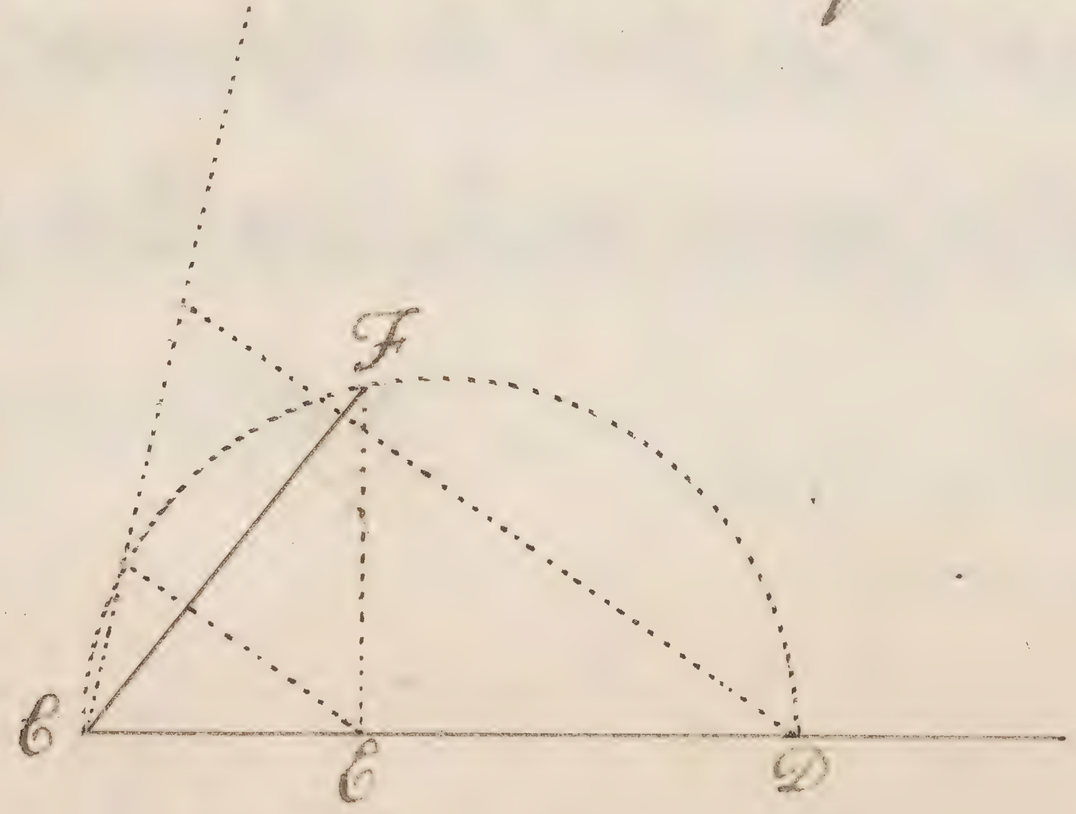
How to enlarge a Line in power according to any proportion assigned.

In fig. 32 let C G be a line given to be enlarged in power as the line B 16 to the Line G 25.

First (by Problem 12) find a line in proportion to the given line C G as B to G which will be C D upon which line describe the Semicircle C F D & on the point C erect the perpendicular C F then draw the line C F which shall be in power to C G as B to G.

B ——— 16

G ——— 25

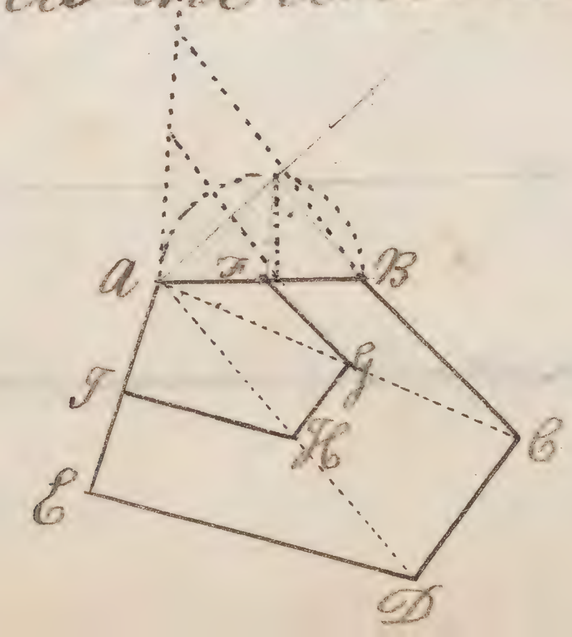


Problem 27.

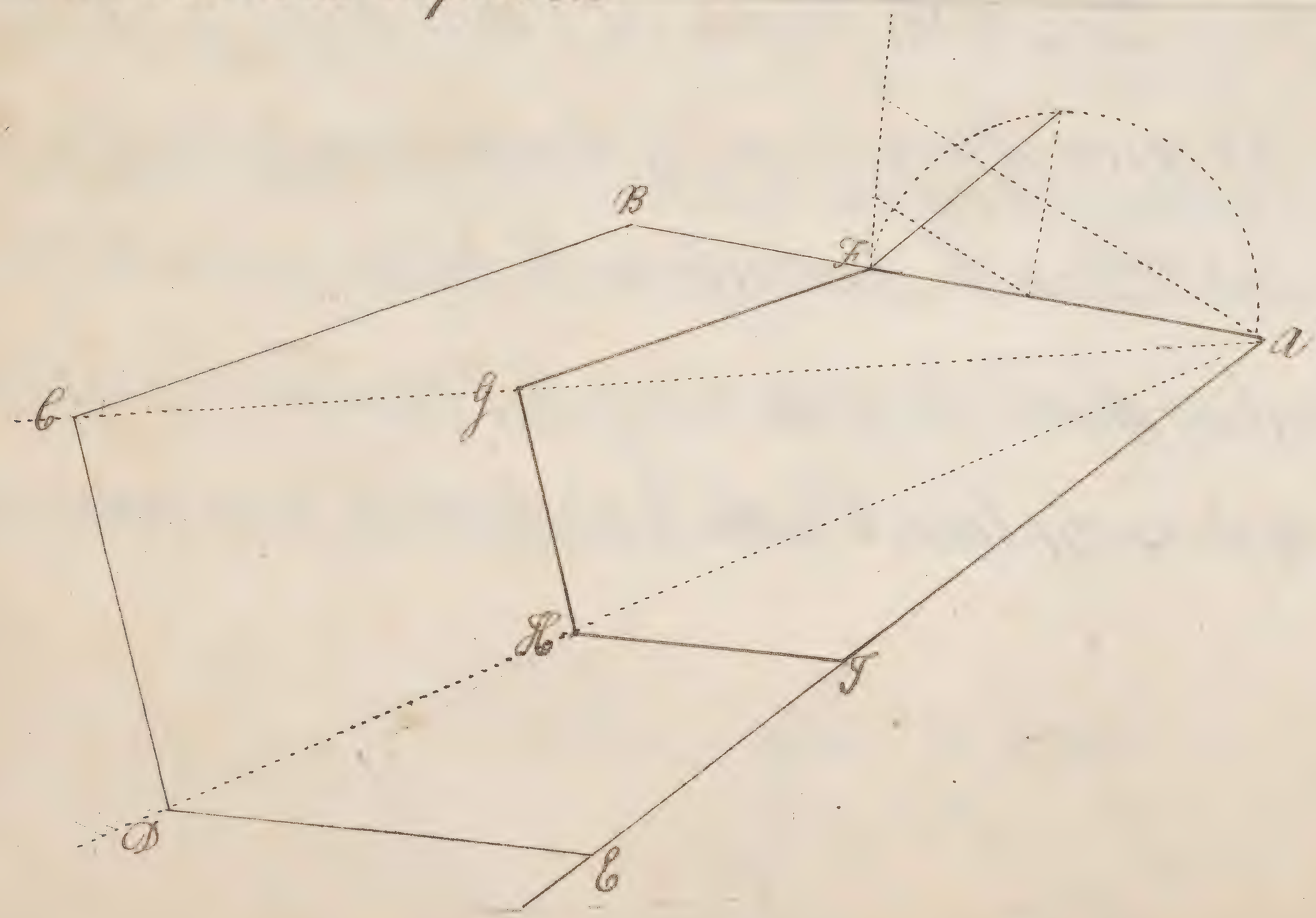
To diminish or enlarge a Plot given according to any Proportion required.

Let ABCDE (fig: 33) be a Plot given to be diminish'd in Power as L to K 5.

Divide one of the Sides (as AB) in Power as L to K in such sort that the Power of A F may be the Power of A B as L to K. Then from the Angle A draw lines to the Points C & D that done by F draw a parallel to B C to cut A C in G as F G. Again from G draw a parallel to D C to cut A D in H. Lastly from H draw a parallel to D E to cut A E in I so shall the Plot A F G H I be alike A B C D E & in proportion to it as the line K to the line L which was required.



Also if the lesser plot were given & it were required to make a greater in proportion to it a K to L. Then from the point A draw the lines AB & AD at length also increase AF & AI that done enlarge AF in power as L to K which set from A to B then by B draw a parallel to Fg to cut Ac in b as Bb; Likewise from b draw a parallel to gH to cut AD in D as bD. Lastly a parallel from D to HI as Dc to cut AI being increased in & so shall you include the plot ABcDE & like AFgHIE in proportion thereunto as the line K is to the line L which was required.



Problem 28.

How to make a Triangle, which shall contain any Number of Acres Roods & Perches & whose Base shall be equal to any (possible) Number given. Vid: Fig: 34.

If it be required to make a Triangle which shall contain 5 Acres, 2 Roods, 30 Perches whose Base shall contain 50 Perches, you must first reduce your 5 Acres, 2 Roods, & 30 Perches all into Perches in this manner.

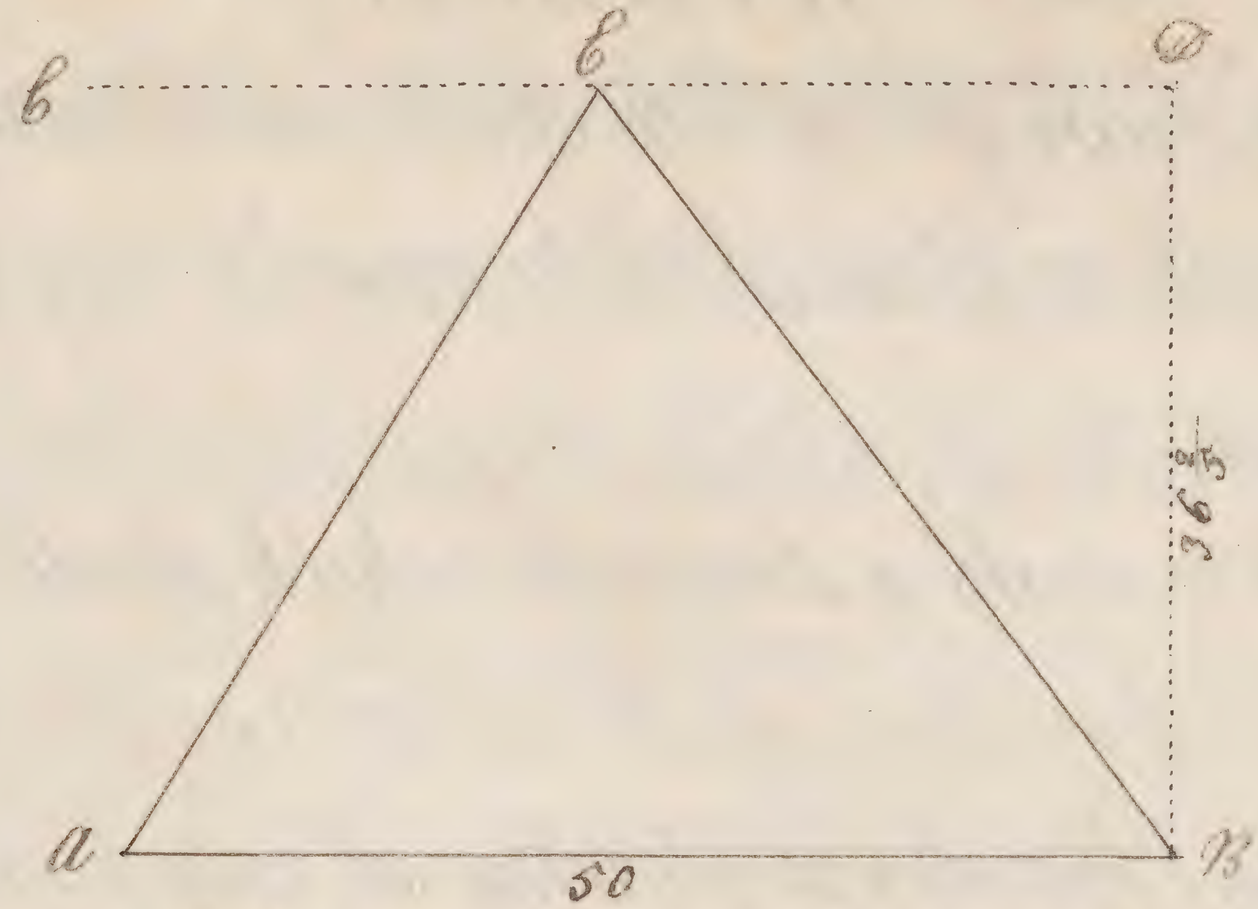
First (because 4 Roods make one Acre) multiply your 5 Acres by 4, which makes 20, to which add the 2 odd Roods so have you 22 Roods in your 5 Acres 2 Roods. Then (because 40 Perches make one Rood) multiply your 22 Roods by 40, which makes 880 Perches, to which add the 30 odd Perches, & you shall have 910, the Number of Perches in 5 Acres, 2 Roods, 30 Perches.

Now, to make a Triangle which shall contain 910 Perches, & whose Base shall be 50 Perches, do thus, you must double the Number of Perches given, Viz 910, & they make 1820. then, because the Base of the triangle must contain 50 Perches, divide 1820 by 50, the Quotient will be 36 2/5, which will be the Length of the Perpendicular of your Triangle. This done, from any equal Scale lay down the line AB equal to 50 Perches: then upon B, raise the perpendicular BD equal to 36 2/5 Perches, & draw the Line CD parallel to AB: then, from any point in the line CD (as from C) draw

the Lines b a & b B including the Triangle a b B, which shall contain 5 Acres, 2 Roods, & 30 Perches which was required.

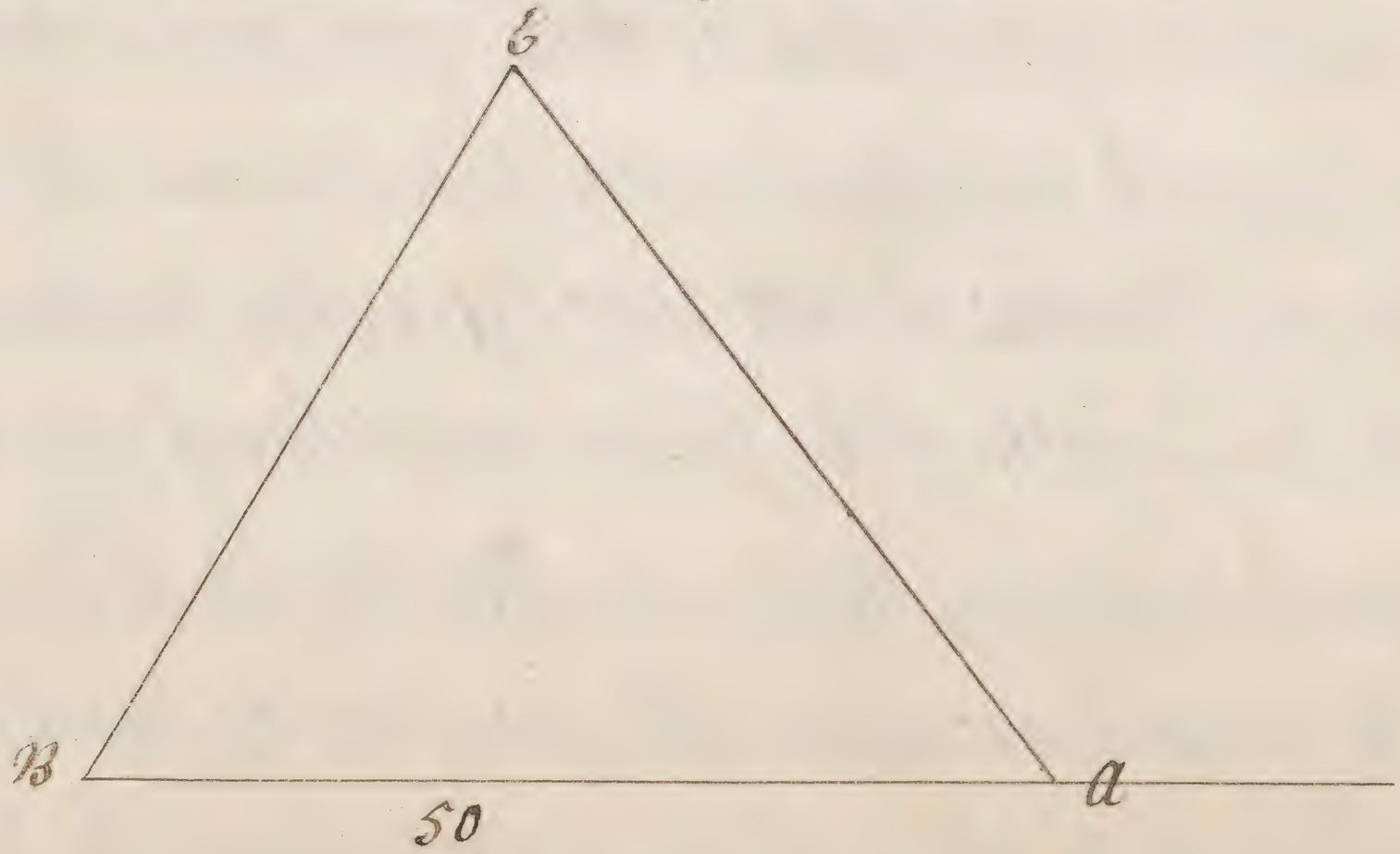
a R P
5 . 2 . 30

4
22
40
910
2
5)1820
$36\frac{2}{5}$



In like manner, if the Lines b B, a B & also the point b were given in position, & it be required to draw from b the line b a, so that the Triangle b a B may contain 910 Perches, & the nearest distance of b from the Line B a be $36\frac{2}{5}$ Perches then divide 1820, the double of 910, & the Quotient 50 gives the length of B a; therefore make B a 50 Perches & draw b a, & so will the Triangle b a B contain as required.

36.4)1820(50
1820

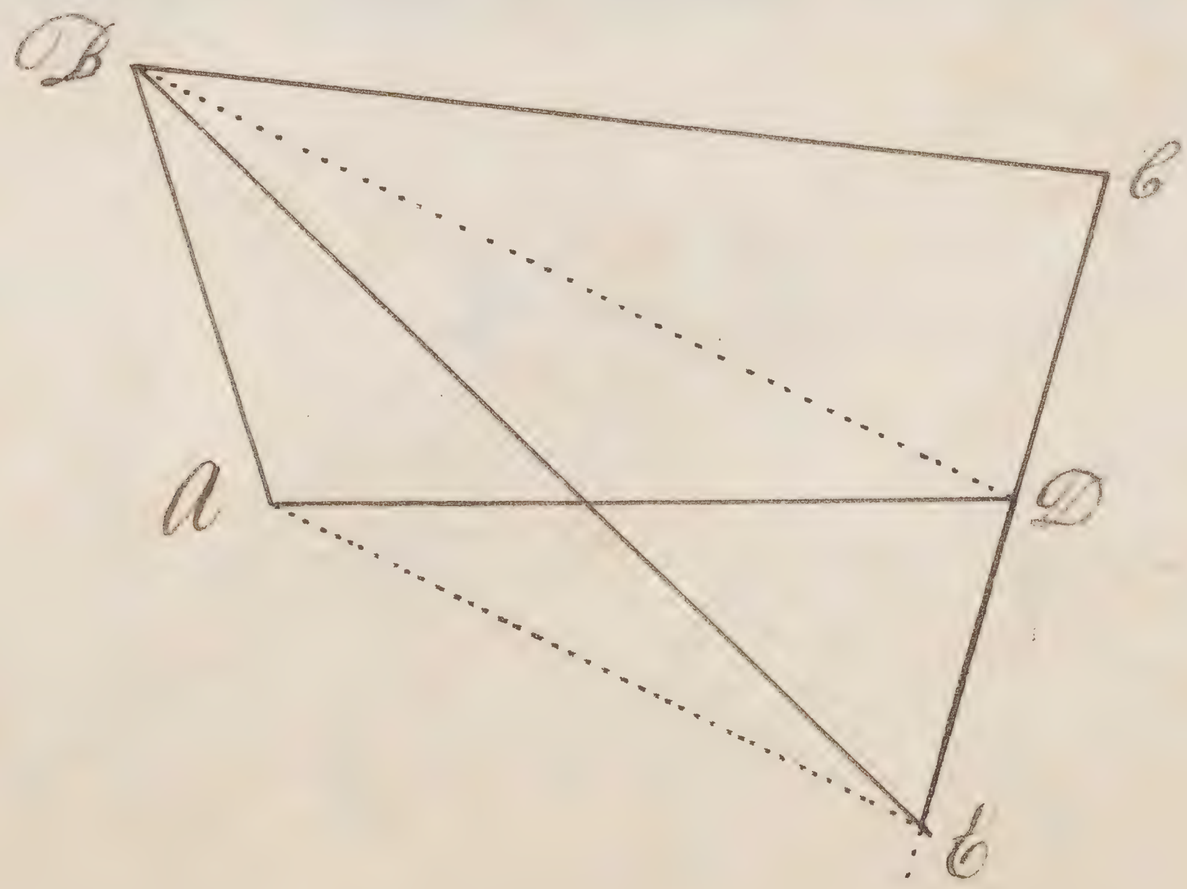


Problem 29.

How to reduce a Trapezia into Triangle by a Line drawn from any Angle thereof.

The Trapezia given is ABCD (fig: 35) & it is required to reduce the same into a Triangle.

First extend the Line DC, & draw the Diagonal BD, then from the point A draw the Line Ae parallel to BD, extending it till it cut the Side CD extended in the point e. Lastly from B, draw the Line Be, constituting the Triangle BCe which shall be equal to the Trapezia ABCD.

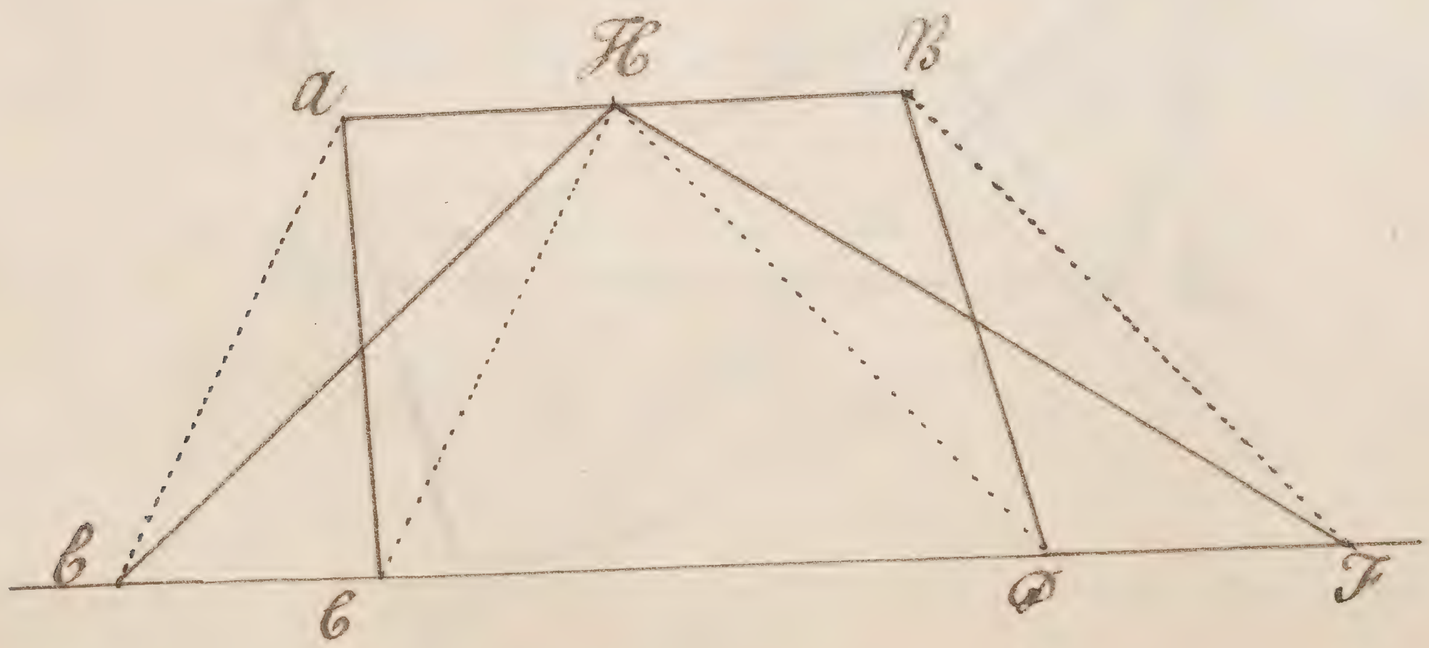


Problem 30.

How to reduce a Trapezia into a Triangle by lines drawn from any point in any of the Sides thereof.

Let ABCD (fig: 36) be a Trapezia given, & let H be a point in one of the Sides thereof, from which point D let it be required to draw Lines, which shall reduce the Trapezia into a Triangle.

First, extend the Side which is opposite to the given point, Viz, the Side CD, both ways to E & F, & farther if need require, & then from the point H, draw Lines to the Angles C & D as the lines HC & HD; also draw the lines AE & BF parallel to HC & HD, cutting the extended line CD in the points E & F. Lastly, if from the point H you draw the Lines HE & HF, you shall constitute the Triangle HEF, which shall be equal to the Trapezia ABCD.

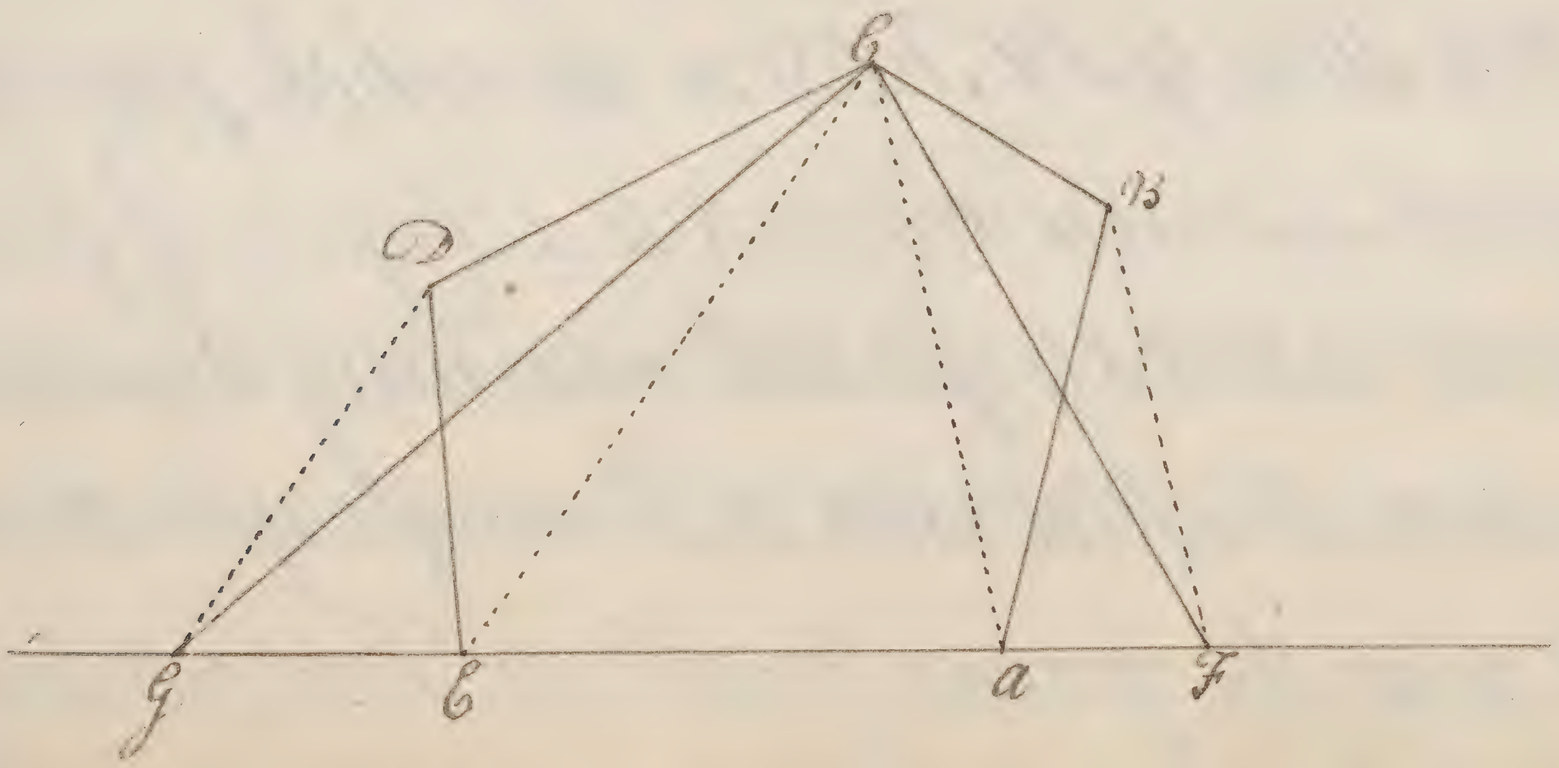


Problem 31.

How to reduce an irregular Plot of five sides into a Triangle.

The irregular plot given is ABCDE (fig: 37) & it is required to reduce the same into a Triangle.

First extend the Side AE both ways to F & G, & from the angle E draw the lines EA & EB to the angles A & B: Then from the point B draw the Line BF parallel to EA cutting the extended Side AE in F: also from the point D draw the Line DG parallel to EB cutting also the extended Side AE in G. Lastly from the angle E draw the Lines EF & EG constituting the Triangle EFG which is equal to the Plot ABCDE.



Problem 32.

How to reduce a Plot of Six Seven or Eight Sides into a Triangle.

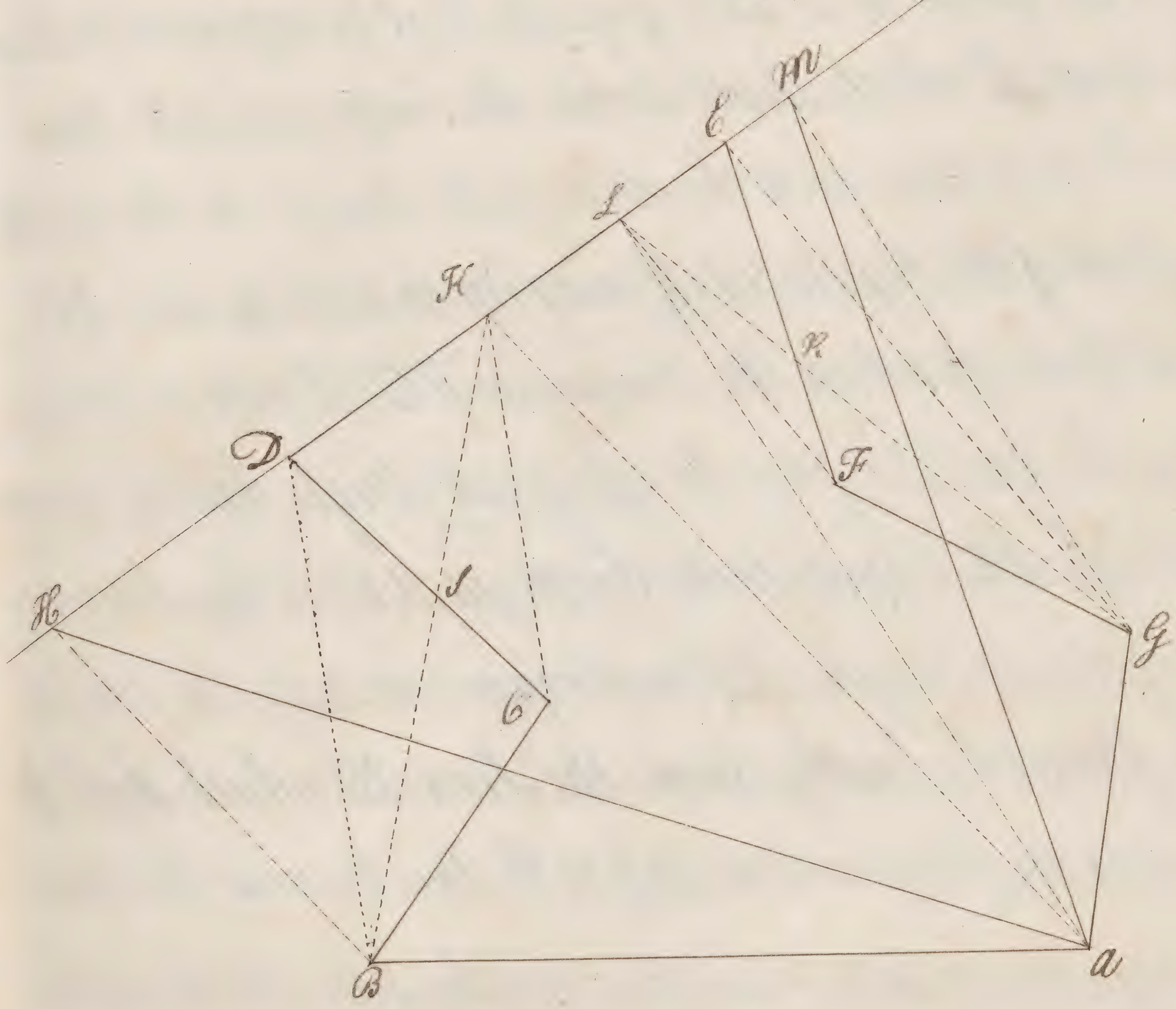
Let ABCDEFG (fig: 30) be an irregular Plot given to be reduced into a Triangle.

Let the Angles E & F in this Figure be within the field because it often comes so in Practice.

First draw the line BD & parallel thereto the Line EK then if you draw the Line BK it will cut off from the Figure the Triangle DKS & will take in the Triangle BES equal thereto & the Side BK will supply the Use of the two Sides BE & ED. Also draw the line GL & parallel thereto the line FL: then if you draw the line GL it shall cut off from the Figure the Triangle LKE & take in the Triangle GKF equal thereto & the Line or Side GL will supply the use of the other two Sides GF & FE & so the whole Plot ABCDEFG (consisting of seven Sides) is reduced to a Figure of five Sides namely into the Figure ABKLG yet still retaining the same Quantity. Now to reduce this Plot into a Triangle you must work in all respects as in the former Problem. First produce the

Side D E of the given figure on both Sides to H & M then draw the Lines a H & a L & parallel to them the Lines B H & g M cutting the line D E (being extended) in H & M. Lastly if you draw the Lines a H & a M you shall constitute the Triangle a H M which shall be equal to the irregular Plot A B C D E F g which was required.

and here note that if the Number of Sides be never so many yet this Way of Reduction will bring them to Triangles.



Problem 33.

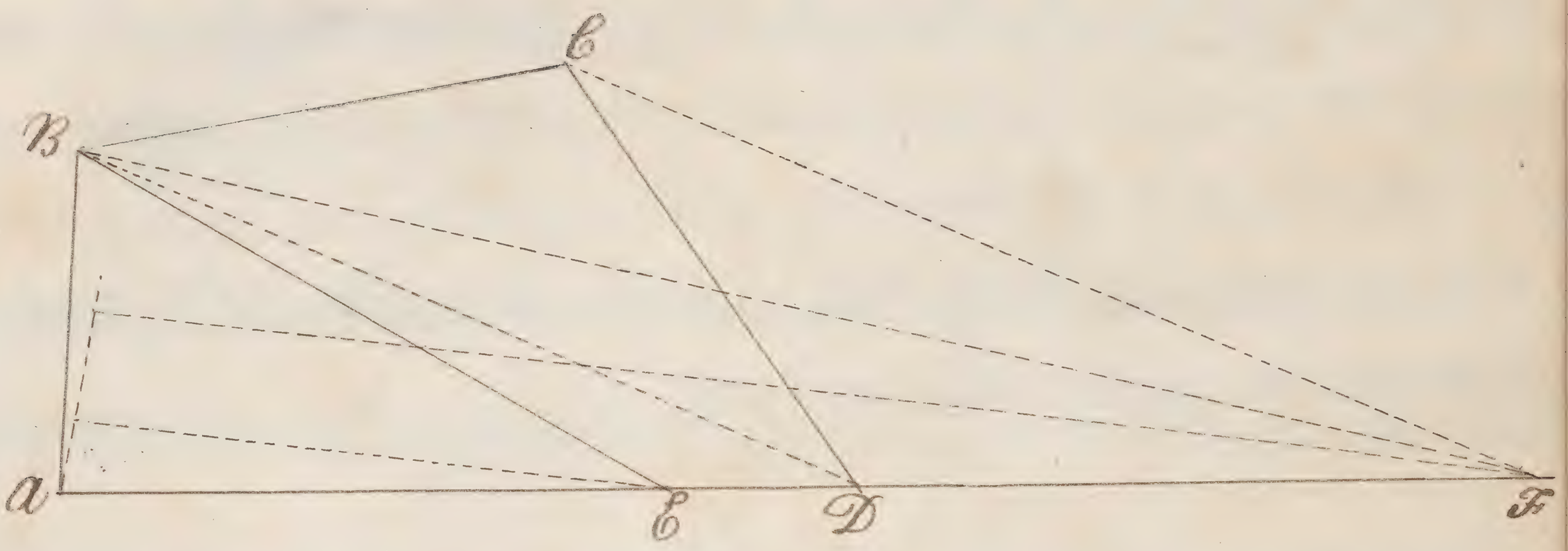
A Trapezia being given how from any Angle thereof to divide the same into two parts being in proportion one to the other as two given right Lines & to set the part cut off towards an assigned Side.

Let the Trapezia given be A B C D (fig 39) & let it be required to draw a line from the Angle B which shall divide the Trapezia into two parts being in proportion one to the other as the line G 2 to the line H 3 & that the lesser part of the Figure cut off may lye towards the Side A B.

First (by Problem 29) reduce the Trapezia A B C D into a Triangle by drawing the line B F from the assigned Angle thereby constituting the Triangle A B F equal to the Trapezia A B C D this done divide the Base of the Triangle A F in proportion as G to H which will be in the point E. Lastly draw the Line B E which shall divide the Trapezia in proportion as G to H. Now because the lesser part of the Trapezia was to be set towards the Side A B therefore the lesser part of the Line must be set from A to E. But as in this Problem so in all those that follow we must continue that line on which

the partition falls in order to reduce it to a Triangle otherwise this Method of parting would not succeed.

Here note that the same manner of working is to be observed if it had been required to divide the Trapezia by a line drawn from any of the other Angles.



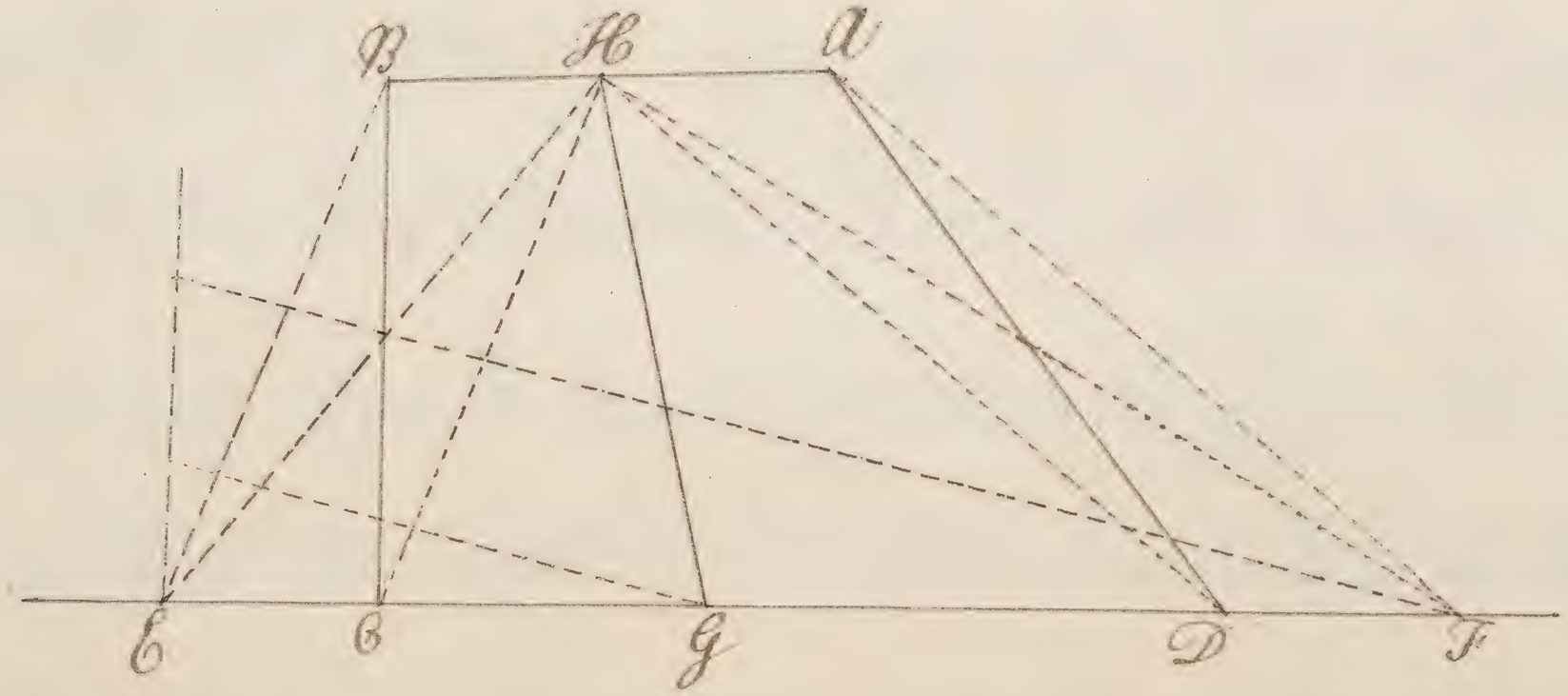
Problem 34.

A Trapezia being given how from a Point given in any Side thereof to draw a line which shall the same into two parts in proportion as two given lines.

The Trapezia given is A B C D (fig. 10) & it is required from the point H to draw a line which shall divide the Trapezia in proportion as 2 to 9.

First prolong the side C D & reduce the whole Trapezia into a Triangle H C F (by Problem 29) then divide the line C F in proportion as 2 to 2 which will fall in the point G therefore draw the line H G which shall divide the Trapezia into two parts in proportion as 2 to 2 which was required.

In this as in the foregoing if the point G had fallen between C & c, the line A C must have been produced instead of D C & then the point G would have fallen between C & a.



Problem 35.

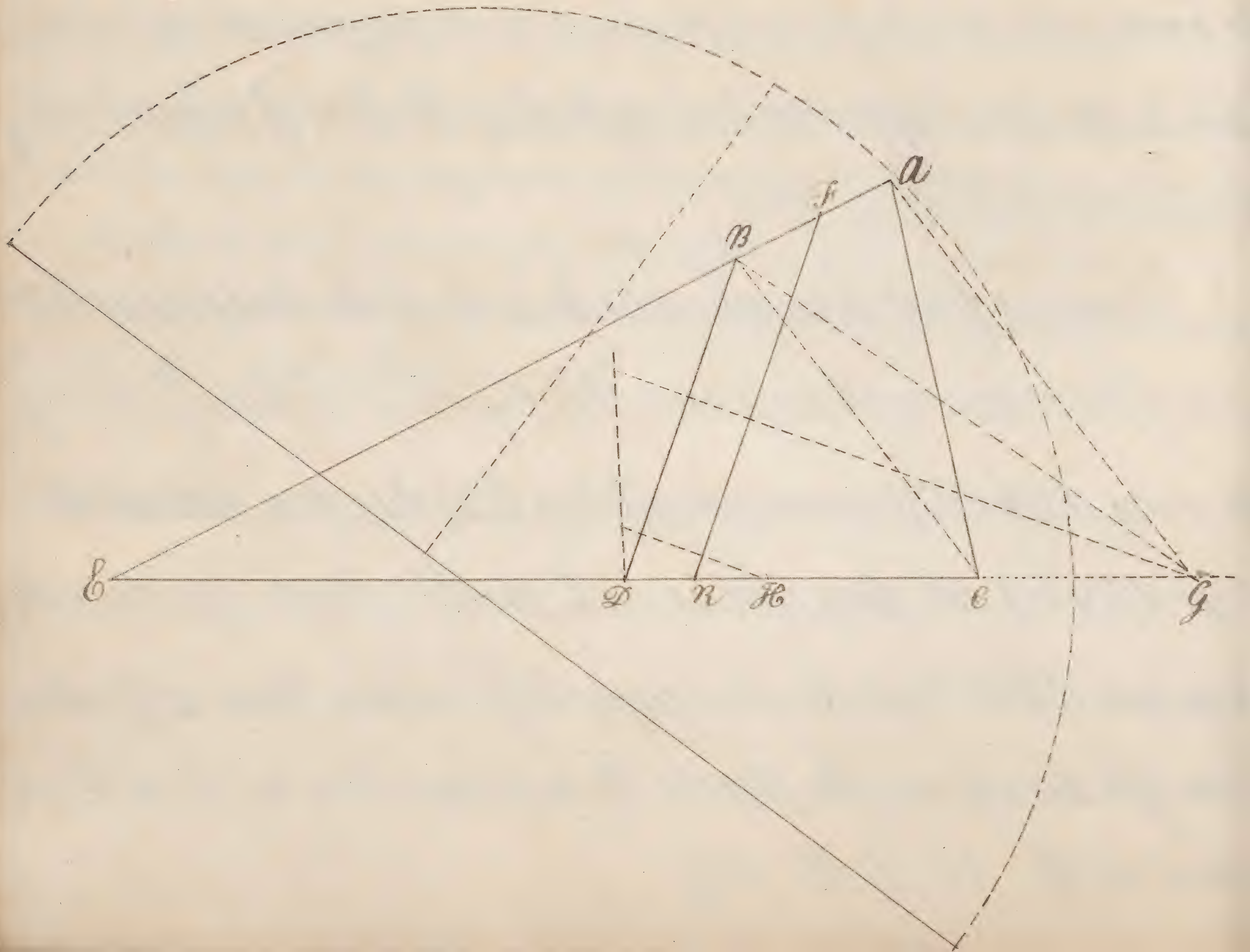
A Trapezia being given how to divide the same into two parts in proportion to two lines given & so that the line of partition may be parallel to any Side thereof.

The Trapezia given is ABCD (fig: 11) & it is required to divide the same into two parts which shall be in proportion one to the other as the Line K(1) to the line L(3) & that the line of partition may be parallel to the Side BD.

Consider first through which Side of the Trapezia the Line of partition will pass as in this figure it will pass through the Sides AB & CD (because parallel to BD) therefore extend the Sides AB & CD till they concur in G; then (by Prob: 29) reduce the Trapezia ABCD into the Triangle BGD whose Base is GD which line GD divide in the Point H in proportion as K to L; so that as K : L :: DH : HG.

This done find a Mean proportional between D & H (by Problem 23) as R. Lastly through this point R draw the Line B F parallel to B D which shall divide the Trapezia into two parts being in proportion one to the other as the Line K is to the line L & with a line parallel to the Side B D which was required.

But if it had been required to divide the Trapezia by a Line drawn parallel to to the Side C D then the lines C A & D B must have been extended but the rest of the Work must be performed as before.



Problem 36.

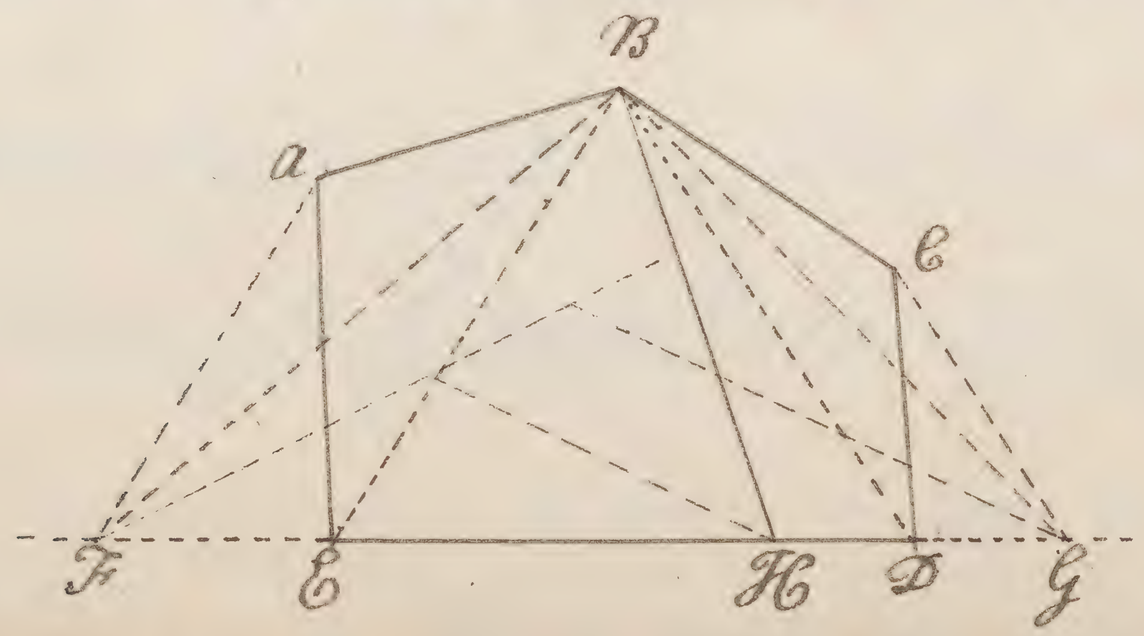
The Figure of a Plot being given how to divide the same into two Parts being in proportion one to the other as two given Lines are with a line drawn from an Angle assigned.

Let A B C D E (fig: 42) represent the plot of a Field or such like & let it be required to divide the same into two Parts being in Proportion one to the other as the Line R 3 to the line S 7 by a line drawn from the Angle B.

First reduce the Plot A B C D E into the Triangle B F G (by Prob: 31) so shall the Line F G be the Base of a Triangle equal to the given Plot then (by Prob: 12) divide this line F G into two Parts in the point H in Proportion one to the other as the line R is to the line S so that as R : S :: G H : H F.

Lastly draw the line B H which shall divide your given Plot into two parts which shall have such Proportion one to the other as the line R to the line S.

If H (in this work) falls between $\left\{\begin{matrix} D \& G \\ E \& F \end{matrix}\right\}$ then the Figure must be reduced to a Triangle by producing $\left\{\begin{matrix} D C \\ a E \end{matrix}\right\}$ & so then G will fall between $\left\{\begin{matrix} D \& C \\ a \& E \end{matrix}\right\}$

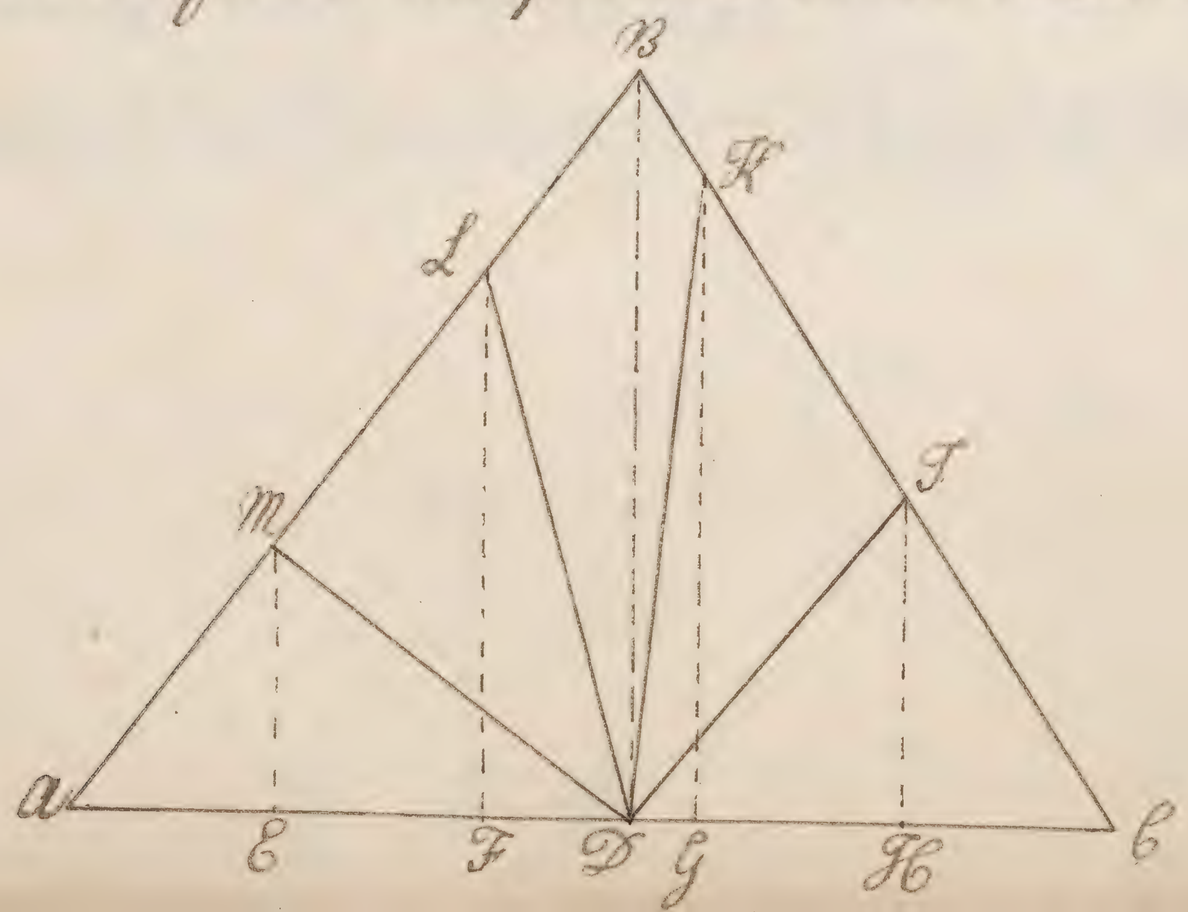


Problem 37.

How to divide a Triangle into any Number of equal parts by lines drawn from a Point given in any Side thereof.

Let it be required to divide the Triangle ABC (fig: 43) into five equal parts by Lines drawn from the Point D.

First from the given Point D to the opposite Angle B draw the line D B then divide the Side of the Triangle A C into five equal parts as E F G & H & through each of these Points Draw lines parallel to D B as E M, F L, G K, & H I then from the Point D draw the Lines D I, D K, D L & D M which shall divide the Triangle ABC into five equal parts from the Point D as was required.



Problem 38.

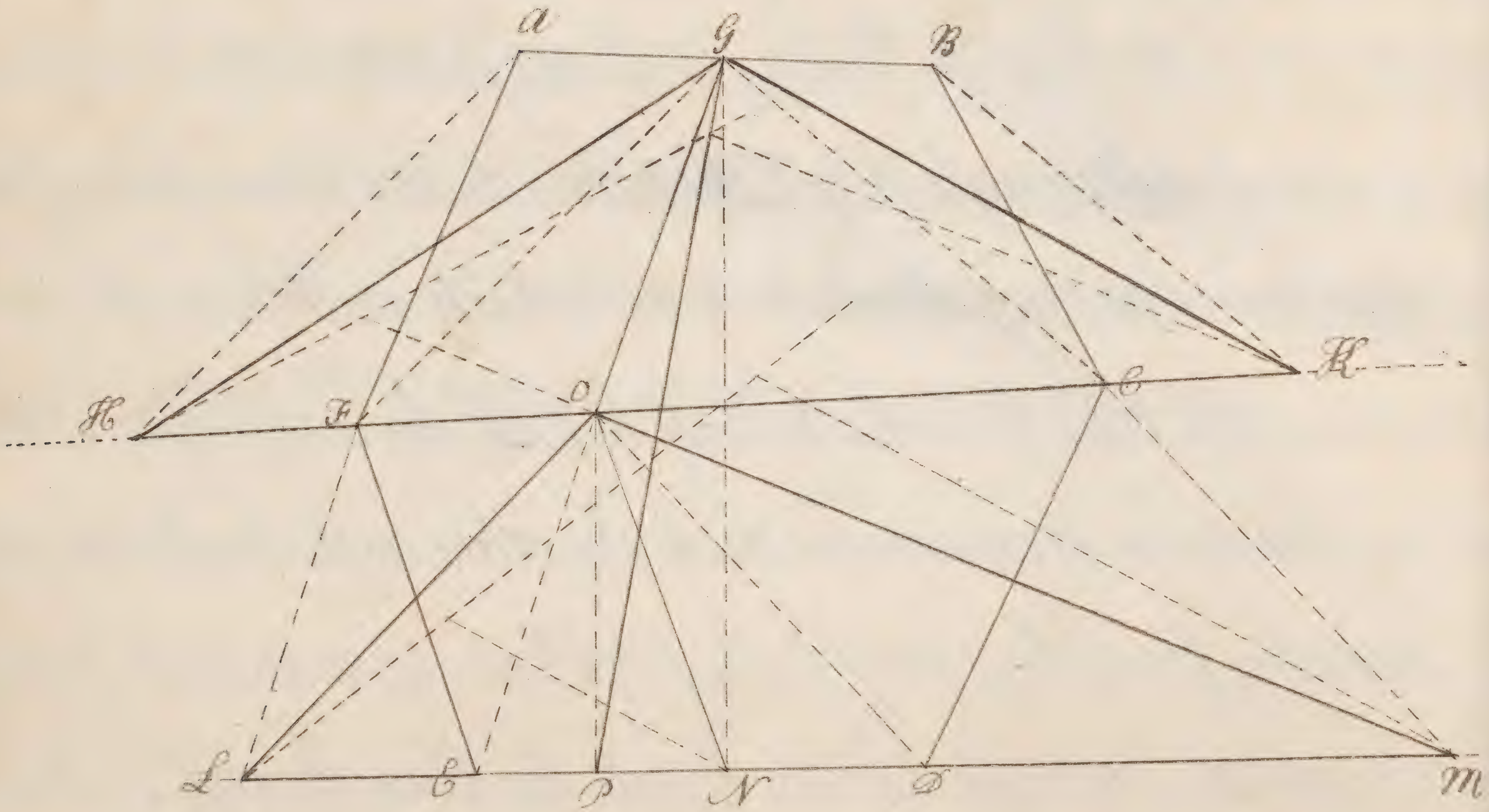
How to divide an irregular Plot of six Sides into two parts according to any assigned proportion by a right Line drawn from a point limited in any of the Sides thereof.

The irregular plot given is ABCDEF (fig: 44) & it is required to divide the same into two parts being in proportion one the other as the line R 2 to the line S 3.

First draw the line AK & (by Problem 29) reduce the Trapezia ABFE into the Triangle AGK then divide the Base thereof AK into two parts in proportion as R to S which will be in the point H then draw the line GO which will divide the Trapezia ABFE into two parts in proportion one to the other as the line R to the lines.

Secondly from the point O (by Problem 29) reduce the Trapezia FEDC into the Triangle OLM & divide the Base thereof LM in the point N in proportion as R to S & draw the Line ON which

will divide the Trapezia F E D into two parts in proportion as R to S & by this means is the whole Plot A B C D & F divided into two parts in proportion as R to S by the lines G O & G N. But it is required to resolve the problem by one right line only drawn from the Point G therefore from the point G draw the line G N & through the point O draw the line e p parallel to G N & lastly from G draw the line G P which shall divide the whole Plot A B C D & F into two parts being in Proportion one to the other as the Line R is to the Line S.



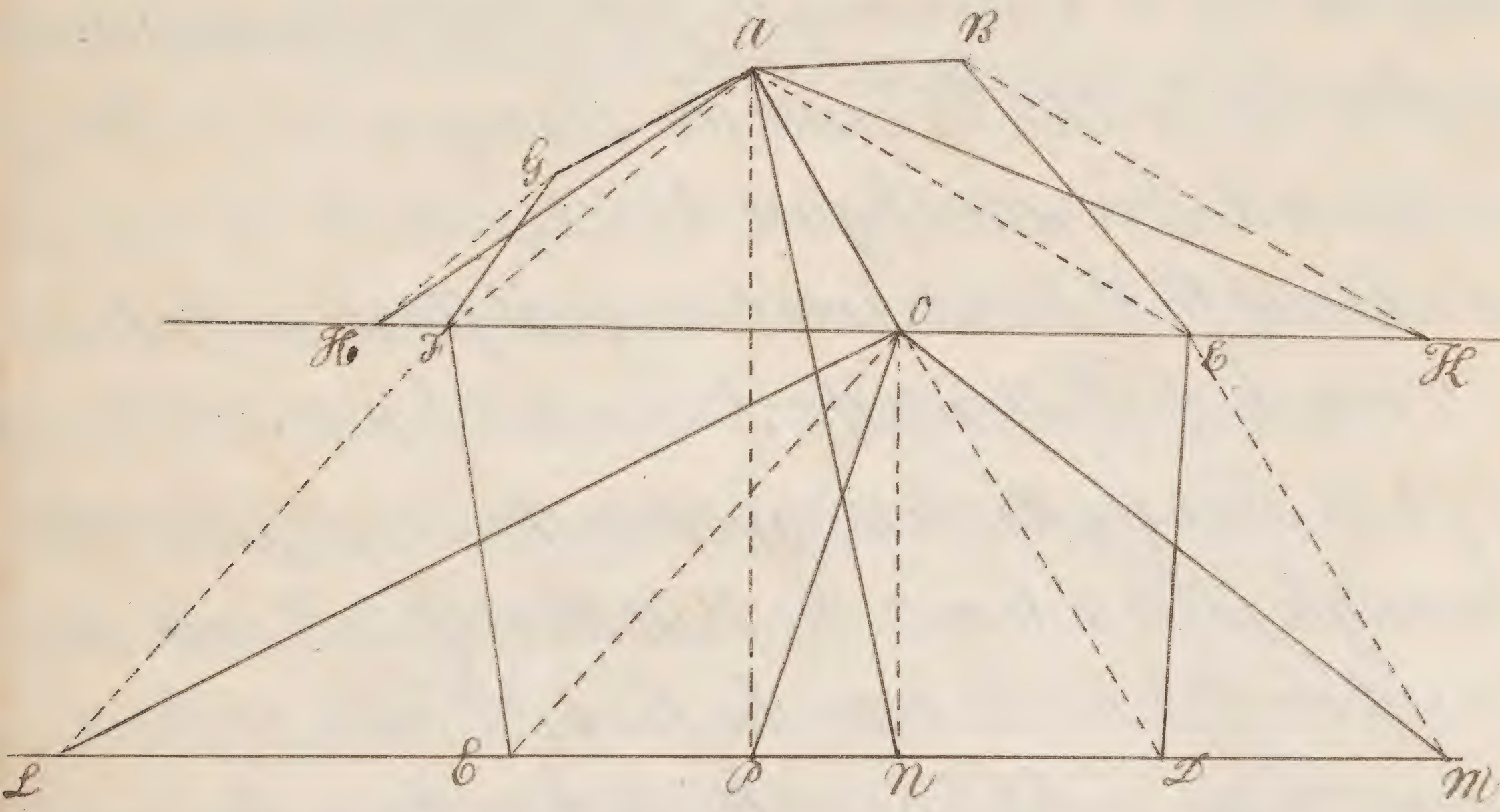
Problem 39.

How to divide an irregular Plot according to any Proportion by a line drawn from any Angle thereof.

Let A B C D E F G (fig. 45) be an irregular Plot, & let it be required to divide the same into two equal parts by a line drawn from the Angle A.

First draw the line F K dividing the Plot into two Parts; namely into the five sided figure A B C F G, & into the Trapezia F C C D, then (by Problem 31) reduce the five sided figure A B C F G into the Triangle K A K, the Base whereof K K divide into two equal parts in O, & draw the Line O A which shall divide the Five sided figure A B C F G into two equal Parts. Then (by Problem 29) reduce the Trapezia F C D E into the Triangle Q E L, & divide the Base thereof L E into two equal parts in the Point N & draw the Line O P; but to perform this Problem by one right line only, do thus; from the point A draw the Line A P and parallel thereunto through the point O draw the Line O N. Lastly if you draw a right line from A to N, it shall divide the whole Plot into two equal parts.

Here note that what has been said concerning the dividing of figures in proportion to right Lines, the same may be effected in Numbers, so that from any Plot you may cut off any Number of Acres Roods & Perches.



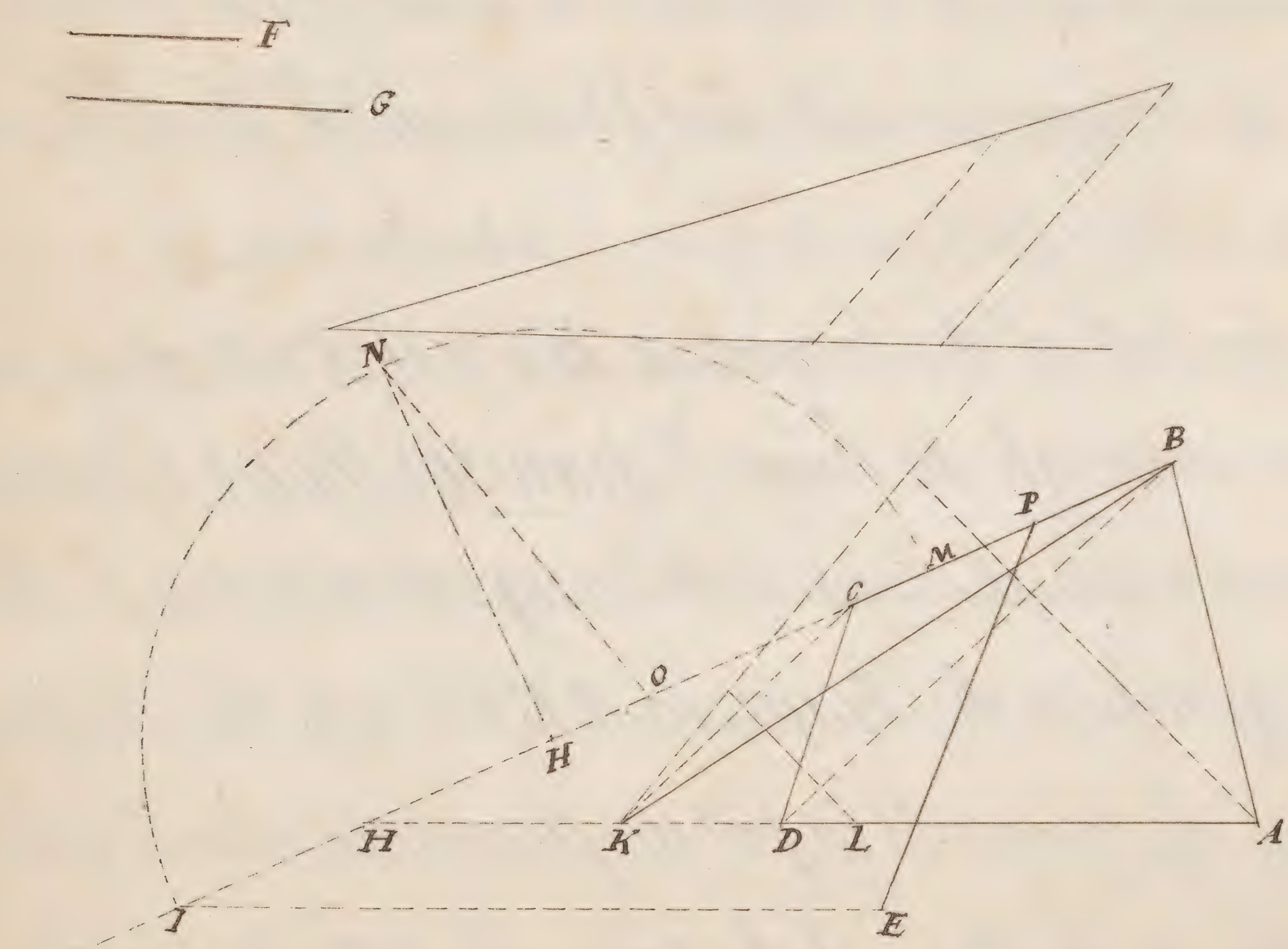
Problem 40.

How to divide a Trapezia into two parts by a line drawn from a Point without, which parts shall be in Proportion one to the other, as two given Lines.

Let the Trapezia given be ABCD (fig. 46) & let the given Point without be E, from which it is required to draw a line which shall divide the Trapezia into two Parts which shall be in proportion one to the other as the line F is to the line G.

Extend the Side of the Trapezia BC & AD till they concur in H: then through the Point E, draw the line EI parallel to AH till meet with the line BH being extended to I, then (by Problem 29) reduce the Trapezia ABCD into the Triangle ABK, & from the Point B let fall the Perpendicular BZ, then (by Prob: 12) divide the Base of the Triangle AK into two Parts in Proportion as F to G which Point of division will fall in L.

This done find a fourth Line which will be in proportion to these three lines I&, H L & H B, that is as I& to H L so H B to H M so is H M the fourth proportional: then (by Problem 23) find a mean proportional between the Lines I H & H M which is H N: then set H N perpendicular upon B H, & divide H M into two equal parts in O then draw the line O N which you shall set from O to N. Lastly if you draw the Line & N, it shall divide the Trapezia A B & D into two parts, which shall be in proportion one to the other as the line F is to the line G.



Problem 41.

From any irregular Plot to cut off therefrom any (possible) Number of Acres, Roods & Perches, & that by a Straight Line drawn from any Angle of the Field.

Let A B C D E F G H K (fig 47) be a Field given containing 15 Acres & 14 Perches and let it be required to cut off from the same field 4 Acres & 12 Perches & that by a right line drawn from the Angle C.

First reduce the 15 Acres, 14 Perches all into Perches (as is taught by Prob: 28) making 2414. Also reduce the 5 Acres and 12 Perches into Perches which make 812.

Secondly from the Angle C draw a right line to any other Angle of the field which may seem most convenient for your purpose as to G, by which line CG, you have cutt off a part of the field, Viz, C D E F G.

Thirdly, find (as is hereafter taught) how many Perches are contained in the part cut off C D E F G which suppose to be 700, but the part of the field to be cut off should contain 812 Perches, too little by

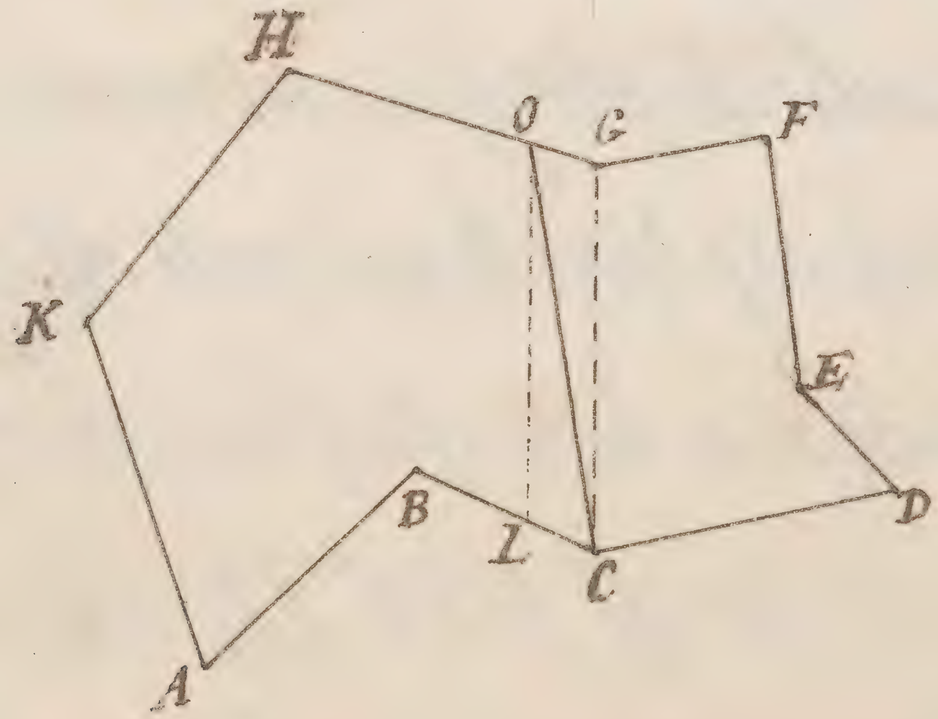
112 perches, & therefore the Line 6 G is not the true line of division but is drawn too much towards the Angle F.

Wherefore to find the true line of Division you must.

Fourthly measure the length of the line 6 G (by the same Scale as you cast up your Plot by) & you shall find it to contain 36 perches in length the half whereof is 18. By which divide 112 (the Difference before found) & the Quotient will be $6\frac{2}{9}$ perches. Wherefore

Fifthly, take $6\frac{2}{9}$ perches from your Scale, & with that distance, from the points 6 & G draw a line parallel to your first supposed line 6 G, as the Line L O, cutting the side of the Plot H G in O.

Lastly draw the right line 6 O & it shall be your true Line of Separation. And so that part of the Plot inclosed by the lines 6 D, D E, 6 F, F G & G O shall contain 812 perches, or 5 Acres & 12 perches. And the other part of the Plot inclosed by the lines 6 B, B A, A K, K H & H O shall contain 1602 perches, or 10 Acres 2 perches.



Trigonometry applicable to Surveying.

Before Mensuration of Triangles & the performance of other usefull & necessary Problems in the Art of Surveying are enter'd upon 'twill be necessary to Shew the Use of the Tables of Sines, Tangents, & Logarithms.

Proposition 1.

any Degree & Minute being given to find the Sine or Tangent thereof.

Seek the Degree & Minute you desire the Sine or Tangent of under the letter D for Degrees & M for Minutes opposite thereto on the right Hand in the next Column under the Word Sine or Tangent you have your Desire.

required the { Sine & Tangent } of 20° | requir'd the { Sine and Tangent } of 10° 30' of 62° 10'

$20^{\circ}\left\{\begin{matrix}\text{Sine } 9.534052\\ \text{Tan: } 9.56107\end{matrix}\right\}$ $40^{\circ}30'\left\{\begin{matrix}\text{Sine } 9.812544\\ \text{Tan: } 9.93150\end{matrix}\right\}$

$62^{\circ}10'\left\{\begin{matrix}\text{Sine } 9.946604\\ \text{Tan: } 10.27738\end{matrix}\right\}$

Proposition 2.

Any Sine or Tangent being given to find the Number of Degrees & Minutes thereunto belonging.

Seek in the second tabular Row among the Sines for the proposed Number and opposite thereto on the Left Hand you will have the Degrees & Minutes corresponding thereunto.

Let 9.866470 be a Sine given to find the degree & Minute of the Quadrant answering thereto.

Ans: $47^{\circ}20''$

Given 9.821264 to find the Arch corresponding.

Ans: $41^{\circ}30'$

Given the Tangent 9.60641 to find the Degrees & Minutes answering thereto.

Ans: 22°

Given 71° 40' to find the Tangent.

Ans: 10.47969

N B. The following Tables being only to every 10th Minute may not exhibit the exact Number proposed tho' they are near enough for any Purpose in Surveying therefore in such case look the nearest in the Table thereto as suppose 9,675059 were given which cannot be exactly found but the nearest in the Table is 9.676328 the Sine of 28° 20' which must be taken instead thereof.

The Table of Logarithms being so plain need no Explanation here.

A Table of Sines & Tangents.

D.M.	Sines	Tangents	D.M.	Sines	Tangents	D.M.	Sines	Tangents
10	7.463726	7.46373	10	8.954499	8.95627	10	9.246775	9.25365
20	7.764754	7.76476	20	8.968249	8.97013	20	9.253761	9.26086
30	7.940842	7.94086	30	8.981573	8.98358	30	9.260633	9.26791
40	8.065776	8.06581	40	8.994497	8.99662	40	9.267395	9.27496
50	8.162681	8.16273	50	9.007044	9.00930	50	9.274049	9.28186
1.00	8.241855	8.24192	6.00	9.019235	9.02162	11.00	9.280599	9.28865
10	8.308794	8.30888	10	9.031089	9.03361	10	9.287048	9.29535
20	8.366777	8.36690	20	9.042625	9.04528	20	9.293399	9.30195
30	8.417919	8.41807	30	9.053859	9.05666	30	9.299655	9.30846
40	8.463665	8.46385	40	9.064806	9.06775	40	9.305819	9.31489
50	8.505045	8.50527	50	9.075480	9.07858	50	9.311893	9.32122
2.00	8.542819	8.54308	7.00	9.085895	9.08914	12.00	9.317879	9.32741
10	8.577566	8.57788	10	9.096062	9.09947	10	9.323780	9.33365
20	8.609734	8.61010	20	9.105992	9.10956	20	9.329599	9.33974
30	8.639679	8.64009	30	9.115698	9.11943	30	9.335337	9.34576
40	8.667689	8.66816	40	9.125187	9.12909	40	9.340996	9.35170
50	8.693998	8.69453	50	9.134470	9.13854	50	9.346579	9.35757
3.00	8.718800	8.71940	8.00	9.143555	9.14780	13.00	9.352088	9.36336
10	8.742259	8.74292	10	9.152451	9.15688	10	9.357524	9.36909
20	8.764511	8.76525	20	9.161164	9.16577	20	9.362889	9.37476
30	8.785675	8.78649	30	9.169702	9.17450	30	9.368185	9.38035
40	8.805852	8.80674	40	9.178072	9.18306	40	9.373414	9.38589
50	8.825130	8.82610	50	9.186280	9.19146	50	9.378577	9.39136
4.00	8.843584	8.84464	9.00	9.194332	9.19971	14.00	9.383675	9.39677
10	8.861283	8.86243	10	9.202235	9.20782	10	9.388711	9.40212
20	8.878285	8.87953	20	9.209992	9.21578	20	9.393685	9.40742
30	8.894643	8.89598	30	9.217609	9.22361	30	9.398600	9.41266
40	8.910404	8.91185	40	9.225092	9.23130	40	9.403455	9.41784
50	8.925609	8.92716	50	9.232444	9.23887	50	9.408254	9.42297
5.00	8.940296	8.94195	10.00	9.239670	9.24632	15.00	9.412996	9.42805

D.M.	Sines	Tan:	D.M.	Sines	Tan:	D.M.	Sines	Tan:
10	9.417684	943308	10	9.537507	956498	10	9.628647	9.67196
20	9.422318	9.43806	20	9.540931	9.56887	20	9.631326	9.67524
30	9.426899	9.44299	30	9.544325	9.57274	30	9.633984	9.67850
40	9.431429	9.44787	40	9.547689	9.57658	40	9.636623	9.68174
50	9.435908	9.45271	50	9.551024	9.58039	50	9.639242	9.68497
16°00	9.440338	9.45750	21°00	9.554329	9.58418	26°00	9.641842	9.68818
10	9.444720	9.46224	10	9.557606	9.58794	10	9.644423	9.69138
20	9.449054	9.46694	20	9.560855	9.59168	20	9.646984	9.69457
30	9.453342	9.47160	30	9.564075	9.59540	30	9.649527	9.69774
40	9.457584	9.47622	40	9.567269	9.59909	40	9.652052	9.70089
50	9.461782	9.48080	50	9.570436	9.60276	50	9.654558	9.70404
17°00	9.465935	9.48534	22°00	9.573575	9.60641	27°00	9.657047	9.70717
10	9.470046	9.48984	10	9.576689	9.61004	10	9.659517	9.71028
20	9.474115	9.49430	20	9.579777	9.61364	20	9.661970	9.71339
30	9.478142	9.49872	30	9.582840	9.61722	30	9.664406	9.71648
40	9.482128	9.50311	40	9.585877	9.62079	40	9.666824	9.71955
50	9.486075	9.50746	50	9.588890	9.62433	50	9.669225	9.72262
18°00	9.489982	9.51178	23°00	9.591878	9.62785	28°00	9.671609	9.72567
10	9.493851	9.51606	10	9.594842	9.63135	10	9.673977	9.72872
20	9.497682	9.52031	20	9.597783	9.63484	20	9.676328	9.73175
30	9.501476	9.52452	30	9.600700	9.63830	30	9.678663	9.73476
40	9.505234	9.52870	40	9.603594	9.64175	40	9.680982	9.73777
50	9.508956	9.53285	50	9.606465	9.64517	50	9.683284	9.74077
19°00	9.512642	9.53697	24°00	9.609313	9.64858	29°00	9.685571	9.74375
10	9.516294	9.54106	10	9.612140	9.65197	10	9.687843	9.74673
20	9.519911	9.54512	20	9.614944	9.65535	20	9.690098	9.74969
30	9.523495	9.54915	30	9.617727	9.65870	30	9.692339	9.75264
40	9.527046	9.55315	40	9.620488	9.66204	40	9.694564	9.75558
50	9.530565	9.55712	50	9.623229	9.66537	50	9.696775	9.75852
20°00	9.534052	9.56107	25°00	9.625948	9.66867	30°00	9.698970	9.76144

D. M.	Sines	Tan:	D. M.	Sines	Tan:	D. M.	Sines	Tan:
10	9.701151	9.76435	10	9.760390	9.84791	10	9809569	9.92638
20	9.703317	9.76726	20	9.762177	9.85059	20	9.811061	9.92894
30	9.705469	9.77015	30	9.763954	9.85327	30	9.812514	9.93150
40	9.707606	9.77303	40	9.765720	9.85594	40	9.814019	9.93406
50	9.709730	9.77591	50	9.767475	9.85860	50	9.815485	9.93661
31°00	9.711839	9.77877	36°00	9.769490	9.86126	41°0	9.816943	9.93916
10	9.713935	9.78163	10	9.770952	9.86391	10	9.818392	9.94171
20	9.716017	9.78448	20	9.772675	9.86656	20	9.819832	9.94426
30	9.718085	9.78732	30	9.774388	9.86921	30	9.821265	9.94681
40	9.720140	9.79015	40	9.776090	9.87185	40	9.822688	9.94935
50	9.722181	9.79297	50	9.777781	9.87448	50	9.824104	9.95190
32°00	9.724210	9.79579	37°0	9.779463	9.87711	42°0	9.825511	9.95444
10	9.726225	9.79860	10	9.781134	9.87974	10	9.826910	9.95698
20	9.728227	9.80140	20	9.782796	9.88236	20	9.828301	9.95952
30	9.730217	9.80419	30	9.784447	9.88498	30	9.829683	9.96205
40	9.732193	9.80697	40	9.786089	9.88759	40	9.831058	9.96459
50	9.734157	9.80975	50	9.787720	9.89020	50	9.832425	9.96712
33°00	9.736109	9.81252	38°0	9.789342	9.89281	43°0	9.833783	9.96966
10	9.738048	9.81528	10	9.790954	9.89541	10	9.835134	9.97219
20	9.739975	9.81803	20	9.792557	9.89801	20	9.836477	9.97472
30	9.741890	9.82078	30	9.794150	9.90060	30	9.837812	9.97725
40	9.743792	9.82352	40	9.795733	9.90320	40	9.839140	9.97978
50	9.745683	9.82626	50	9.797307	9.90578	50	9.840459	9.98231
34°00	9.747562	9.82899	39°0	9.798872	9.90837	44°0	9.841771	9.98484
10	9.749429	9.83171	10	9.800427	9.91095	10	9.843076	9.98736
20	9.751284	9.83442	20	9.801973	9.91353	20	9.844372	9.98989
30	9.753128	9.83713	30	9.803510	9.91610	30	9.845662	9.99242
40	9.754960	9.83984	40	9.805038	9.91868	40	9.846944	9.99495
50	9.756782	9.84254	50	9.806557	9.92125	50	9.848218	9.99747
35°00	9.758591	9.84523	40°0	9.808067	9.92381	45°0	9.849485	10.00000

D. M.	Sines	Tan:	D. M.	Sines	Tan:	D. M.	Sines	Tan:
10	9.850745	10.00253	10	9.885311	10.07875	10	9.914246	10.15747
20	9.851997	10.00505	20	9.886362	10.08132	20	9.915123	10.16016
30	9.853242	10.00758	30	9.887406	10.08390	30	9.915994	10.16287
40	9.854480	10.01011	40	9.888444	10.08647	40	9.916859	10.16558
50	9.855710	10.01263	50	9.889476	10.08905	50	9.917719	10.16829
46 0	9.856934	10.01516	51 0	9.890503	10.09163	56 0	9.918574	10.17101
10	9.858150	10.01769	10	9.891522	10.09422	10	9.919424	10.17374
20	9.859360	10.02022	20	9.892536	10.09680	20	9.920268	10.17648
30	9.860562	10.02275	30	9.893544	10.09939	30	9.921107	10.17922
40	9.861758	10.02528	40	9.894546	10.10199	40	9.921940	10.18197
50	9.862946	10.02781	50	9.895542	10.10459	50	9.922768	10.18472
47 0	9.864127	10.03034	52 0	9.896532	10.10719	57 0	9.923591	10.18748
10	9.865302	10.03288	10	9.897516	10.10980	10	9.924409	10.19025
20	9.866470	10.03541	20	9.898494	10.11241	20	9.925222	10.19303
30	9.867631	10.03795	30	9.899467	10.11502	30	9.926029	10.19581
40	9.868785	10.04048	40	9.900433	10.11764	40	9.926831	10.19860
50	9.869933	10.04302	50	9.901394	10.12026	50	9.927628	10.20140
48 0	9.871073	10.04556	53 0	9.902349	10.12289	58 0	9.928420	10.20421
10	9.872208	10.04810	10	9.903298	10.12552	10	9.929207	10.20703
20	9.873335	10.05065	20	9.904241	10.12815	20	9.929989	10.20985
30	9.874456	10.05319	30	9.905179	10.13079	30	9.930766	10.21268
40	9.875571	10.05574	40	9.906111	10.13344	40	9.931537	10.21552
50	9.876678	10.05829	50	9.907037	10.13609	50	9.932304	10.21837
49 0	9.877780	10.06084	54 0	9.907958	10.13874	59 0	9.933066	10.22123
10	9.878875	10.06339	10	9.908873	10.14140	10	9.933822	10.22409
20	9.879963	10.06594	20	9.909782	10.14406	20	9.934574	10.22697
30	9.881045	10.06850	30	9.910686	10.14673	30	9.935320	10.22985
40	9.882121	10.07106	40	9.911854	10.14941	40	9.936062	10.23274
50	9.883191	10.07362	50	9.912477	10.15209	50	9.936799	10.23565
50 0	9.884254	10.07619	55 0	9.913364	10.15477	60 0	9.937531	10.23856

D. M.	Sines	Tan:	D. M.	Sines	Tan:	D. M.	Sines	Tan:
10	9.938258	10.24148	10	9.957863	10.33463	10	9.973443	10.44288
20	9.938980	10.24442	20	9.958415	10.33796	20	9.973897	10.44685
30	9.939697	10.24736	30	9.959023	10.34130	30	9.974347	10.45085
40	9.940409	10.25031	40	9.959596	10.34465	40	9.974792	10.45488
50	9.941117	10.25327	50	9.960165	10.34803	50	9.975233	10.45894
61 0	9.941819	10.25625	66 0	9.960730	10.35142	71 0	9.975670	10.46303
10	9.942517	10.25923	10	9.961290	10.35483	10	9.976103	10.46715
20	9.943210	10.26223	20	9.961846	10.35825	20	9.976532	10.47130
30	9.943898	10.26524	30	9.962398	10.36170	30	9.976957	10.47548
40	9.944582	10.26825	40	9.962945	10.36516	40	9.977377	10.47969
50	9.945261	10.27128	50	9.963488	10.36865	50	9.977794	10.48391
62 0	9.945935	10.27433	67 0	9.964026	10.37215	72 0	9.978206	10.48822
10	9.946604	10.27738	10	9.964560	10.37567	10	9.978615	10.49254
20	9.947269	10.28044	20	9.965090	10.37921	20	9.979019	10.49689
30	9.947929	10.28352	30	9.965615	10.38278	30	9.979419	10.50128
40	9.948584	10.28661	40	9.966136	10.38636	40	9.979816	10.50570
50	9.949235	10.28972	50	9.966653	10.38996	50	9.980208	10.51016
63 0	9.949881	10.29283	68 0	9.967166	10.39359	73 0	9.980596	10.51466
10	9.950522	10.29596	10	9.967674	10.39724	10	9.980980	10.51920
20	9.951159	10.29911	20	9.968178	10.40091	20	9.981361	10.52378
30	9.951791	10.30226	30	9.968678	10.40460	30	9.981737	10.52839
40	9.952419	10.30543	40	9.969173	10.40832	40	9.982109	10.53305
50	9.953042	10.30862	50	9.969665	10.41206	50	9.982477	10.53776
64 0	9.953660	10.31182	69 0	9.970152	10.41582	74 0	9.982842	10.54250
10	9.954274	10.31503	10	9.970635	10.41961	10	9.983202	10.54729
20	9.954883	10.31826	20	9.971113	10.42342	20	9.983558	10.55213
30	9.955488	10.32150	30	9.971588	10.42726	30	9.983910	10.55701
40	9.956088	10.32476	40	9.972058	10.43113	40	9.984259	10.56194
50	9.956684	10.32804	50	9.972524	10.43502	50	9.984603	10.56692
65 0	9.957276	10.33133	70 0	9.972986	10.43893	75 0	9.984944	10.57195

D. M.	Sines	Tan:	D. M.	Sines	Tan:	D. M.	Sines	Tan:
10	9.985280	10.57703	10	9.993572	10.76113	10	9.998453	11.07284
20	9.985613	10.58216	20	9.993789	10.76870	20	9.998558	11.08815
30	9.985942	10.58734	30	9.994003	10.77639	30	9.998659	11.10402
40	9.986266	10.59258	40	9.994212	10.78422	40	9.998757	11.12047
50	9.986587	10.59788	50	9.994418	10.79218	50	9.998851	11.13757
76 0	9.986904	10.60323	81 0	9.994620	10.80029	86 0	9.998941	11.15536
10	9.987217	10.60864	10	9.994818	10.80854	10	9.999027	11.17390
20	9.987526	10.61411	20	9.995013	10.81694	20	9.999110	11.19326
30	9.987832	10.61965	30	9.995203	10.82550	30	9.999189	11.21351
40	9.988133	10.62524	40	9.995390	10.83423	40	9.999265	11.23475
50	9.988430	10.63091	50	9.995573	10.84312	50	9.999336	11.25708
77 0	9.988724	10.63664	82 0	9.995753	10.85220	87 0	9.999404	11.28060
10	9.989014	10.64243	10	9.995928	10.86146	10	9.999469	11.30547
20	9.989299	10.64830	20	9.996100	10.87091	20	9.999529	11.33184
30	9.989582	10.65424	30	9.996269	10.88057	30	9.999586	11.35991
40	9.989860	10.66026	40	9.996433	10.89044	40	9.999640	11.38990
50	9.990134	10.66635	50	9.996594	10.90053	50	9.999689	11.42212
78 0	9.990404	10.67253	83 0	9.996751	10.91086	88 0	9.999735	11.45692
10	9.990671	10.67878	10	9.996904	10.92142	10	9.999778	11.49473
20	9.990934	10.68512	20	9.997053	10.93225	20	9.999816	11.53615
30	9.991193	10.69154	30	9.997199	10.94334	30	9.999851	11.58193
40	9.991448	10.69805	40	9.997341	10.95472	40	9.999882	11.63310
50	9.991699	10.70465	50	9.997480	10.96639	50	9.999910	11.69112
79 0	9.991947	10.71135	84 0	9.997614	10.97838	89 0	9.999934	11.75808
10	9.992190	10.71814	10	9.997745	10.99070	10	9.999954	11.83727
20	9.992430	10.72504	20	9.997873	11.00338	20	9.999971	11.93419
30	9.992666	10.73203	30	9.997996	11.01642	30	9.999983	12.05914
40	9.992898	10.73914	40	9.998116	11.02987	40	9.999993	12.23524
50	9.993127	10.74635	50	9.998232	11.04373	50	9.999998	12.53627
80 0	9.993351	10.75368	85 0	9.998344	11.05805	90 0	10.000000	Infinite.

The Table of Logarithms.

n.	Log:	n.	Log:	n.	Log:	n	Log:	n.	Log:
1	0.000000	26	1.414973	51	1.707570	76	1.880814	101	2.004321
2	0.301030	27	1.431364	52	1.716003	77	1.886491	102	2.008600
3	0.477121	28	1.447158	53	1.724276	78	1.892095	103	2.012837
4	0.602060	29	1.462398	54	1.732394	79	1.897627	104	2.017033
5	0.698970	30	1.477121	55	1.740363	80	1.903090	105	2.021189
6	0.778151	31	1.491362	56	1.748188	81	1.908485	106	2.025306
7	0.845098	32	1.505150	57	1.755875	82	1.913814	107	2.029384
8	0.903090	33	1.518514	58	1.763428	83	1.919078	108	2.033424
9	0.954242	34	1.531479	59	1.770852	84	1.924279	109	2.037426
10	1.000000	35	1.544068	60	1.778151	85	1.929419	110	2.041393
11	1.041393	36	1.556302	61	1.785330	86	1.934498	111	2.045323
12	1.079181	37	1.568202	62	1.792392	87	1.939519	112	2.049218
13	1.113943	38	1.579784	63	1.799341	88	1.944483	113	2.053078
14	1.146128	39	1.591065	64	1.806180	89	1.949390	114	2.056905
15	1.176091	40	1.602060	65	1.812913	90	1.954242	115	2.060698
16	1.204120	41	1.612784	66	1.819544	91	1.959041	116	2.064458
17	1.230449	42	1.623249	67	1.826075	92	1.963788	117	2.068186
18	1.255272	43	1.633468	68	1.832509	93	1.968483	118	2.071882
19	1.278754	44	1.643453	69	1.838849	94	1.973128	119	2.075547
20	1.301030	45	1.653212	70	1.845098	95	1.977724	120	2.079181
21	1.322219	46	1.662758	71	1.851258	96	1.982271	121	2.082785
22	1.342423	47	1.672098	72	1.857332	97	1.986772	122	2.086360
23	1.361728	48	1.681241	73	1.863323	98	1.991226	123	2.089905
24	1.380211	49	1.690196	74	1.869232	99	1.995635	124	2.093422
25	1.397940	50	1.698970	75	1.875061	100	2.000000	125	2.096910

n.	Log:	n.	Log:	n	Log:	n.	Log:	n.	Log:
126	2.100371	156	2,193125	186	2.269513	216	2.334454	246	2.390935
127	2.103804	157	2.195900	187	2.271842	217	2.336459	247	2.392697
128	2.107210	158	2.198657	188	2.274158	218	2.338456	248	2.394452
129	2.110590	159	2.201397	189	2.276462	219	2.340444	249	2.396199
130	2.113943	160	2.204120	190	2.278754	220	2.342423	250	2.397940
131	2.117271	161	2.206826	191	2.281033	221	2.344392	251	2.399674
132	2.120574	162	2.209515	192	2.283301	222	2.346353	252	2.401401
133	2.123852	163	2.212188	193	2.285557	223	2.348305	253	2.403121
134	2.127105	164	2.214844	194	2.287802	224	2.350248	254	2.404834
135	2.130334	165	2.217484	195	2.290035	225	2.352183	255	2.406540
136	2.133539	166	2.220108	196	2.292256	226	2.354108	256	2.408240
137	2.136721	167	2.222716	197	2.294466	227	2.356026	257	2.409933
138	2.139879	168	2.225309	198	2.296665	228	2.357935	258	2.411620
139	2.143015	169	2.227887	199	2.298853	229	2.359836	259	2.413299
140	2.146128	170	2.230449	200	2.301030	230	2.361728	260	2.414973
141	2.149219	171	2.232996	201	2.303196	231	2.363612	261	2.416641
142	2.152288	172	2.235528	202	2.305351	232	2.365488	262	2.418301
143	2.155336	173	2.238046	203	2.307496	233	2.367356	263	2.419956
144	2.158362	174	2.240549	204	2.309630	234	2.369216	264	2.421604
145	2.161368	175	2.243038	205	2.311754	235	2.371068	265	2.423246
146	2.164353	176	2.245513	206	2.313867	236	2.372912	266	2.424882
147	2.167317	177	2.247973	207	2.315970	237	2.374748	267	2.426511
148	2.170262	178	2.250420	208	2.318063	238	2.376577	268	2.428135
149	2.173186	179	2.252853	209	2.320146	239	2.378398	269	2.429752
150	2.176091	180	2.255273	210	2.322219	240	2.380211	270	2.431364
151	2.178977	181	2.257679	211	2.324282	241	2.382017	271	2.432969
152	2.181844	182	2.260071	212	2.326336	242	2.383815	272	2.434569
153	2.184691	183	2.262451	213	2.328379	243	2.385606	273	2.436163
154	2.187521	184	2.264818	214	2.330414	244	2.387390	274	2.437751
155	2.190332	185	2.267172	215	2.332438	245	2.389166	275	2.439333

n.	Log.	n.	Log.	n.	Log.	n.	Log	n.	Log.
276	2.440909	311	2.492760	346	2.539076	381	2.580925	416	2.619093
277	2.442479	312	2.494155	347	2.540329	382	2.582063	417	2.620136
278	2.441045	313	2.495544	348	2.541579	383	2.583199	418	2.621176
279	2.445604	314	2.496929	349	2.542825	384	2.584331	419	2.622214
280	2.447158	315	2.498311	350	2.544068	385	2.585461	420	2.623249
281	2.448706	316	2.499687	351	2.545307	386	2.586587	421	2.624282
282	2.450249	317	2.501059	352	2.546543	387	2.587711	422	2.625312
283	2.451786	318	2.502427	353	2.547775	388	2.588832	423	2.626340
284	2.453318	319	2.503791	354	2.549003	389	2.589949	424	2.627366
285	2.454845	320	2.505150	355	2.550228	390	2.591065	425	2.628389
286	2.456366	321	2.506505	356	2.551450	391	2.592177	426	2.629409
287	2.457882	322	2.507856	357	2.552668	392	2.593286	427	2.630428
288	2.459392	323	2.509203	358	2.553883	393	2.594393	428	2.631444
289	2.460898	324	2.510545	359	2.555094	394	2.595496	429	2.632457
290	2.462398	325	2.511883	360	2.556303	395	2.596597	430	2.633468
291	2.463893	326	2.513218	361	2.557507	396	2.597695	431	2.634477
292	2.465383	327	2.514548	362	2.558709	397	2.598790	432	2.635484
293	2.466868	328	2.515874	363	2.559907	398	2.599883	433	2.636488
294	2.468347	329	2.517196	364	2.561101	399	2.600973	434	2.637489
295	2.469822	330	2.518514	365	2.562293	400	2.602060	435	2.638489
296	2.471292	331	2.519828	366	2.563481	401	2.603144	436	2.639486
297	2.472756	332	2.521138	367	2.564666	402	2.604226	437	2.640481
298	2.474216	333	2.522444	368	2.565848	403	2.605305	438	2.641474
299	2.475671	334	2.523746	369	2.567026	404	2.606381	439	2.642465
300	2.477121	335	2.525045	370	2.568202	405	2.607455	440	2.643453
301	2.478566	336	2.526339	371	2.569374	406	2.608526	441	2.644439
302	2.480007	337	2.527630	372	2.570543	407	2.609594	442	2.645422
303	2.481443	338	2.528917	373	2.571709	408	2.610660	443	2.646404
304	2.482874	339	2.530199	374	2.572872	409	2.611723	444	2.647383
305	2.484299	340	2.531479	375	2.574031	410	2.612784	445	2.648360
306	2.485721	341	2.532754	376	2.575188	411	2.613842	446	2.649335
307	2.487138	342	2.534026	377	2.576341	412	2.614897	447	2.650308
308	2.488551	343	2.535294	378	2.577492	413	2.615950	448	2.651278
309	2.489958	344	2.536558	379	2.578639	414	2.617000	449	2.652246
310	2.491362	345	2.537819	380	2.579784	415	2.618048	450	2.653213

n.	Log:	n.	Log:	n.	Log:	n.	Log:	n.	Log:
451	2.654177	486	2.686636	521	2.716838	556	2.745075	591	2.771587
452	2.655138	487	2.687529	522	2.717671	557	2.745855	592	2.772322
453	2.656098	488	2.688419	523	2.718502	558	2.746634	593	2.773055
454	2.657056	489	2.689309	524	2.719331	559	2.747412	594	2.773786
455	2.658011	490	2.690196	525	2.720159	560	2.748188	595	2.774517
456	2.658965	491	2.691081	526	2.720986	561	2.748963	596	2.775246
457	2.659916	492	2.691965	527	2.721811	562	2.749736	597	2.775974
458	2.660865	493	2.692847	528	2.722634	563	2.750508	598	2.776701
459	2.661813	494	2.693727	529	2.723456	564	2.751279	599	2.777427
460	2.662758	495	2.694605	530	2.724276	565	2.752048	600	2.778151
461	2.663701	496	2.695482	531	2.725095	566	2.752816	601	2.778874
462	2.664642	497	2.696356	532	2.725912	567	2.753583	602	2.779596
463	2.665581	498	2.697229	533	2.726727	568	2.754348	603	2.780317
464	2.666518	499	2.698101	534	2.727541	569	2.755112	604	2.781037
465	2.667453	500	2.698970	535	2.728354	570	2.755875	605	2.781755
466	2.668386	501	2.699838	536	2.729165	571	2.756636	606	2.782473
467	2.669317	502	2.700704	537	2.729974	572	2.757396	607	2.783189
468	2.670246	503	2.701568	538	2.730782	573	2.758155	608	2.783904
469	2.671173	504	2.702431	539	2.731589	574	2.758912	609	2.784617
470	2.672098	505	2.703291	540	2.732394	575	2.759668	610	2.785329
471	2.673021	506	2.704151	541	2.733197	576	2.760422	611	2.786041
472	2.673942	507	2.705008	542	2.733999	577	2.761176	612	2.786751
473	2.674861	508	2.705864	543	2.734799	578	2.761928	613	2.787460
474	2.675778	509	2.706718	544	2.735599	579	2.762679	614	2.788168
475	2.676694	510	2.707570	545	2.736397	580	2.763428	615	2.788875
476	2.677607	511	2.708421	546	2.737193	581	2.764176	616	2.789581
477	2.678518	512	2.709270	547	2.737987	582	2.764923	617	2.790285
478	2.679428	513	2.710117	548	2.738781	583	2.765669	618	2.790988
479	2.680336	514	2.710963	549	2.739572	584	2.766413	619	2.791691
480	2.681241	515	2.711807	550	2.740363	585	2.767156	620	2.792392
481	2.682145	516	2.712649	551	2.741152	586	2.767898	621	2.793092
482	2.683047	517	2.713491	552	2.741939	587	2.768638	622	2.793790
483	2.683947	518	2.714329	553	2.742725	588	2.769377	623	2.794488
484	2.684845	519	2.715167	554	2.743509	589	2.770115	624	2.795185
485	2.685742	520	2.716003	555	2.744293	590	2.770852	625	2.795880

n.	Log:	n.	Log:	n.	Log:	n.	Log:	n.	Log:
626	2.796574	661	2.820201	696	2.842609	731	2.863917	766	2.884229
627	2.797268	662	2.820858	697	2.843233	732	2.864511	767	2.884795
628	2.797959	663	2.821514	698	2.843855	733	2.865104	768	2.885361
629	2.798651	664	2.822168	699	2.844477	734	2.865696	769	2.885926
630	2.799341	665	2.822822	700	2.845098	735	2.866287	770	2.886491
631	2.800029	666	2.823474	701	2.845718	736	2.866878	771	2.887054
632	2.800717	667	2.824126	702	2.846337	737	2.867467	772	2.887617
633	2.801404	668	2.824776	703	2.846955	738	2.868056	773	2.888179
634	2.802089	669	2.825426	704	2.847573	739	2.868643	774	2.888741
635	2.802774	670	2.826075	705	2.848189	740	2.869232	775	2.889302
636	2.803457	671	2.826723	706	2.848805	741	2.869818	776	2.889862
637	2.804139	672	2.827369	707	2.849419	742	2.870404	777	2.890421
638	2.804821	673	2.828015	708	2.850033	743	2.870989	778	2.890979
639	2.805501	674	2.828660	709	2.850646	744	2.871573	779	2.891537
640	2.806180	675	2.829304	710	2.851258	745	2.872156	780	2.892095
641	2.806858	676	2.829947	711	2.851869	746	2.872739	781	2.892651
642	2.807535	677	2.830589	712	2.852480	747	2.873321	782	2.893207
643	2.808211	678	2.831229	713	2.853089	748	2.873902	783	2.893762
644	2.808886	679	2.831869	714	2.853698	749	2.874482	784	2.894316
645	2.809559	680	2.832509	715	2.854306	750	2.875061	785	2.894869
646	2.810233	681	2.833147	716	2.854913	751	2.875640	786	2.895423
647	2.810904	682	2.833784	717	2.855519	752	2.876218	787	2.895975
648	2.811575	683	2.834421	718	2.856124	753	2.876795	788	2.896526
649	2.812245	684	2.835056	719	2.856728	754	2.877371	789	2.897077
650	2.812913	685	2.835691	720	2.857332	755	2.877947	790	2.897627
651	2.813581	686	2.836324	721	2.857935	756	2.878522	791	2.898176
652	2.814248	687	2.836957	722	2.858537	757	2.879096	792	2.898725
653	2.814913	688	2.837588	723	2.859138	758	2.879669	793	2.899273
654	2.815578	689	2.838219	724	2.859739	759	2.880242	794	2.899821
655	2.816241	690	2.838849	725	2.860338	760	2.880814	795	2.900367
656	2.816904	691	2.839478	726	2.860937	761	2.881385	796	2.900913
657	2.817565	692	2.840106	727	2.861534	762	2.881955	797	2.901458
658	2.818226	693	2.840733	728	2.862131	763	2.882525	798	2.902003
659	2.818885	694	2.841359	729	2.862728	764	2.883093	799	2.902547
660	2.819544	695	2.841985	730	2.863323	765	2.883661	800	2.903090

N.	Log:	N.	Log:	N.	Log:	N.	Log:	N.	Log
801	2.903633	837	2.922725	873	2.941014	909	2.958564	945	2.975432
802	2.904174	838	2.923244	874	2.941511	910	2.959041	946	2.975891
803	2.904716	839	2.923762	875	2.942008	911	2.959518	947	2.976350
804	2.905256	840	2.924279	876	2.942504	912	2.959995	948	2.976808
805	2.905796	841	2.924796	877	2.942999	913	2.960471	949	2.977266
806	2.906335	842	2.925312	878	2.943493	914	2.960946	950	2.977724
807	2.906874	843	2.925828	879	2.943989	915	2.961421	951	2.978181
808	2.907411	844	2.926342	880	2.944483	916	2.961895	952	2.978637
809	2.907949	845	2.926857	881	2.944976	917	2.962369	953	2.979093
810	2.908485	846	2.927370	882	2.945469	918	2.962843	954	2.979548
811	2.909021	847	2.927883	883	2.945961	919	2.963315	955	2.980003
812	2.909556	848	2.928396	884	2.946452	920	2.963788	956	2.980458
813	2.910091	849	2.928908	885	2.946943	921	2.964259	957	2.980912
814	2.910624	850	2.929419	886	2.947434	922	2.964731	958	2.981366
815	2.911158	851	2.929929	887	2.947924	923	2.965202	959	2.981819
816	2.911690	852	2.930439	888	2.948413	924	2.965672	960	2.982271
817	2.912222	853	2.930949	889	2.948902	925	2.966142	961	2.982723
818	2.912753	854	2.931458	890	2.949390	926	2.966611	962	2.983175
819	2.913284	855	2.931966	891	2.949878	927	2.967079	963	2.983626
820	2.913814	856	2.932474	892	2.950365	928	2.967548	964	2.984077
821	2.914343	857	2.932981	893	2.950852	929	2.968016	965	2.984527
822	2.914872	858	2.933487	894	2.951338	930	2.968483	966	2.984977
823	2.915399	859	2.933993	895	2.951823	931	2.968949	967	2.985426
824	2.915927	860	2.934498	896	2.952308	932	2.969416	968	2.985875
825	2.916454	861	2.935003	897	2.952792	933	2.969882	969	2.986324
826	2.916980	862	2.935507	898	2.953276	934	2.970347	970	2.986772
827	2.917506	863	2.936011	899	2.953759	935	2.970812	971	2.987219
828	2.918030	864	2.936514	900	2.954243	936	2.971276	972	2.987666
829	2.918555	865	2.937016	901	2.954725	937	2.971739	973	2.988113
830	2.919078	866	2.937518	902	2.955207	938	2.972203	974	2.988559
831	2.919601	867	2.938019	903	2.955688	939	2.972666	975	2.989005
832	2.920123	868	2.938519	904	2.956168	940	2.973128	976	2.989449
833	2.920645	869	2.939019	905	2.956649	941	2.973589	977	2.989895
834	2.921166	870	2.939519	906	2.957128	942	2.974051	978	2.990339
835	2.921686	871	2.940018	907	2.957607	943	2.974512	979	2.990783
836	2.922206	872	2.940516	908	2.958086	944	2.974972	980	2.991226

n.	Log:	n.	Log:	n.	Log:	n.	Log:	n.	Log:
981	2.991669	985	2.993436	989	2.995196	993	2.996949	997	2.998695
982	2.992112	986	2.993877	990	2.995635	994	2.997386	998	2.999131
983	2.992554	987	2.994317	991	2.996074	995	2.997823	999	2.999565
984	2.992995	988	2.994757	992	2.996512	996	2.998259	1000	3.000000

Theoretick Considerations &c.

1 A Triangle is a figure consisting of three Sides and three Angles

2 Any two Sides of a Triangle are called the Sides of the angle comprehended by them.

3 The Measure of an Angle is the Quantity of an Arch of a Circle described on the angular Point, & cutting both the containing Sides of the same Angle.

4 A Degree is the 360.th part of any Circle therefore a Semicircle contains 180° & a Quadrant 90.°

5 The Complement of an Angle less than a Quadrant is what that Angle wants of 90.°

6 The Supplement of an Arch or Angle to a Semicircle is the Remainder thereof to 180.°

7 A Triangle is either right or oblique angled.

8 A Rectangled Triangle is that which hath one right Angle of 90.° and the two remaining Angles thereof are oblique.

9 In every right angled Triangle that Line which subtends the right Angle is called the Hypothenuse and the shorter of the other two Lines is commonly called the Perpendicular or Normal Line and the longer the Base.

10 In every rectangular Triangle if you have one of the Acute Angles given the other is also given it being the Complement to 90.°

11 The three Angles of any rectilineal Triangle whatsoever are equal to two Rectangles so that if you have any two of the Angles given you have the third also given it being the Supplement of the other two to 180.°

12 In all plain Triangles whatsoever the Sides are in Proportion one to another as the Sines of the Angles opposite to those Sides.

Triangles resolved.

Which is done either by the Tables of Sines and Logarithms or by the Lines of artificial Numbers Sines & Tangents on the Index of your Table when any three of the six parts of a Triangle are given to find a fourth.

The several Cases of a Rectangled Triangle will be appear when they are employed in the taking of Heights. And the oblique-angled Triangle for taking of Distances.

Case 1.st

In a rectangled Triangle the Base and the Angle at the Base being given to find the Perpendicular.

Ex.

In the right angled Triangle A, B, C, (fig:1) there is given the Base thereof B, A 400 Feet and the Angle at the Base C, B, A, 30,° & it is requir'd to find the Perpendicular C, A.

Analogy.

As the Sine of the angle B, C, A, 60.°

Is to the Logarithm of the Side B A 400 feet.

So is the Sine of the angle C, A, B, 30.°

To the Logarithm of the Side B C.

A General Rule.

In all Analogies or Proportions whatsoever, wrought by Sines Tangents & Logarithms you must observe this for a general Rule Namely To add the second & third Numbers together & from their aggregate to subtract the first Number so shall the Remainder answer the Question demanded.

How to perform the same Work by the Lines of Sines & Numbers.

But before that can be done Numeration on the Line of numbers must be explained thus.

From the left hand to the Right hand are Divisions of the longer Sort numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2, 3, 4, 5, 6, 7, 8, 9, 1. so that the Beginning, End, & Middle are numbered 1: & whatsoever Value is set on the first one the second or middlemost is 10 times so much; & last one an 100 times so much; & whatsoever Value is set on the {first / middle-most} 1, the same Value is set on the numbered Divisions immediately following the {first / middle-most} 1. that is if the first 1 be called One then shall the {2 / 3 / 4 &c} immediately following it be called {two / three / four &c.} & if the first 1 be call'd 10 then the {2 / 3 / 4 &c.} immediately following it be called {twenty / thirty / forty &c.} And so if the first one be called 100 then the {2 / 3 / 4 &c.} immediately following it shall be called {200 / 300 / 400 &c.} & so on. And in like manner if the middlemost 1 be call'd 10 (which is when the first 1 is call'd One) then the {2 / 3 / 4 &c.} immediately following the middle-

most 1 is call'd {twenty / thirty / forty / &c.} And if the middlemost 1 is call'd 100 (which is when the first 1 is call'd 10) then the {2 / 3 / 4 / &c.} immediately following the middlemost 1 is call'd {200 / 300 / 400 / &c.} & so on.

Secondly between every two of these figur'd Divisions are 10, (shorter than the primary or figur'd one's) & call'd Secondary Divisions. And the value of any Secondary is just 10th part of that Primary one which immediately precedes it: So if a Primary Division be valued {hundreds / tens / unites} the secondary ones immediately following it are {tens / unites / 10th parts} So if the primary Divisions were {hundreds / tens / unites} then the fifth or longest Division between 384 signifies {350 / 35 / 3.5}.

Lastly between these Secondary Divisions are other shorter ones call'd tertiary; in some places between two secondary ones (as those immediately following the middle figures) & in this place the Value of the tertiary ones are each 10th of the Value of the preceding secondary; & in some places 5 between every secondary (as those immediate-

by following the latter figure of 2): And here the Value of the tertiary ones are 2/10 ths of the Values of the preceding secondary: And so if we imagine one single Division between each of these last tertiary ones every one of them will be 1/10 th of their secondary ones. In some places between the secondary is only one tertiary whose Value is 5/10 ths of its preceding secondary; the other tertiary ones here are to be supplied in the Imagination as before.

Therefore to find upon the Line 143 make the first 1 ten & the middlemost 1, an hundred; then of the go on towards 2 count till you come to the 4.th secondary Division which gives the Number 140. lastly count forwards three tertiary Divisions & there will be the Place of 143.

In like manner to find the Number 247; call the middlemost 1 an 100 & so the two following it 200; then go on four of the secondary which will give the Place of 240; lastly go on still onwards to 3.5 of the tertiary & you will have the Place of 247

And here note or observe that the Divisions following the middlemost 1 are best to signify 3 Places of figures but those that immediately follow the first 1 to signify a Number of 1 or 2 Places.

The Line of Sines.

Begin a little below one Degree & run with 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50, 60, 70, 80, 90, And that the Values are not mutable as on the line of Numbers but fixt by the figures annexed.

The secondary Divisions on the left hand of 10 denote the Parts of a Degree & where there are $\left\{\begin{matrix}12\\6\\4\\3\\2\end{matrix}\right\}$ each signifies $\left\{\begin{matrix}5\\10\\15\\20\\30\end{matrix}\right\}$ Min: of a Degree. The secondary Divisions from 10 upwards (where they are in Number 10 between thier Primary) signify whole Degrees & the tertiary ones between them denote parts of a Degree

Where there are but 5 secondary Divisions (as sometimes there are no more between 70 & 80) each Division denotes 2 Degrees.

Between 80 & 90 there generally is only one Division which is the 85.th Degree.

The Line of Tangents begins as the Line of Sines doth & goes on to 45 Degrees. the Degrees above 45 are supplied by those belon 45 & usually numbered backwards. the Places here are just as on the Sines & have not changeable Values as on the Line of Numbers.

For as in the preceding Example the Proportion was.

As the Sine of the Angle 60.°
Is to the Logarithm of 400 feet
So is the Sine of the Angle 30.°
To the Logarithm of the requir'd Side.

Therefore if you work by this General Rule set one foot of the Compasses in the first Term of the Analogy & extend the other to the second term the same Extent shall reach the same way upwards or downwards from the third term in the Proportion to the fourth proportional Number sought. Or

If you set one foot in the first term & extend the other to the third term the same Extent shall reach from the second Term to the fourth also.

s.60° : L:400 :: s.30° : L231

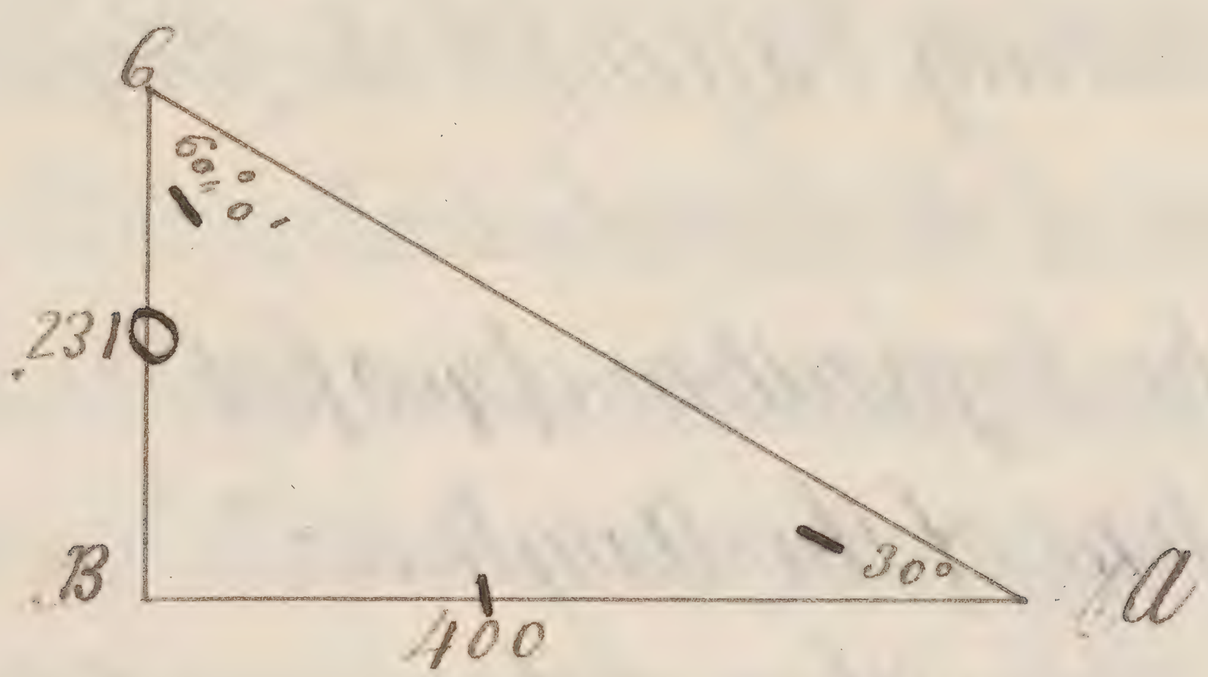
9.937531 : 2.602060 :: 9.698970

2.602060

12.301030

9.937531

231 = 2.363499



The compasses extended from 60° to 30° on the Lines reach from 400 to 231 on the Numbers.

Case 2.

The Base and the Angle at the Base be-ing given to find the Hypothenusal.

In the same triangle A B C, fig: 1, let there be gi-ven (as before) the Base A B 400 feet & the Angle A B C 30° and let it be required to find the Hypothenusal a c.

1st. Analogy by the Tables.

2nd. By the Lines of Sines & Numbers

s. 60° : L: 400 :: s. 90° : L 462

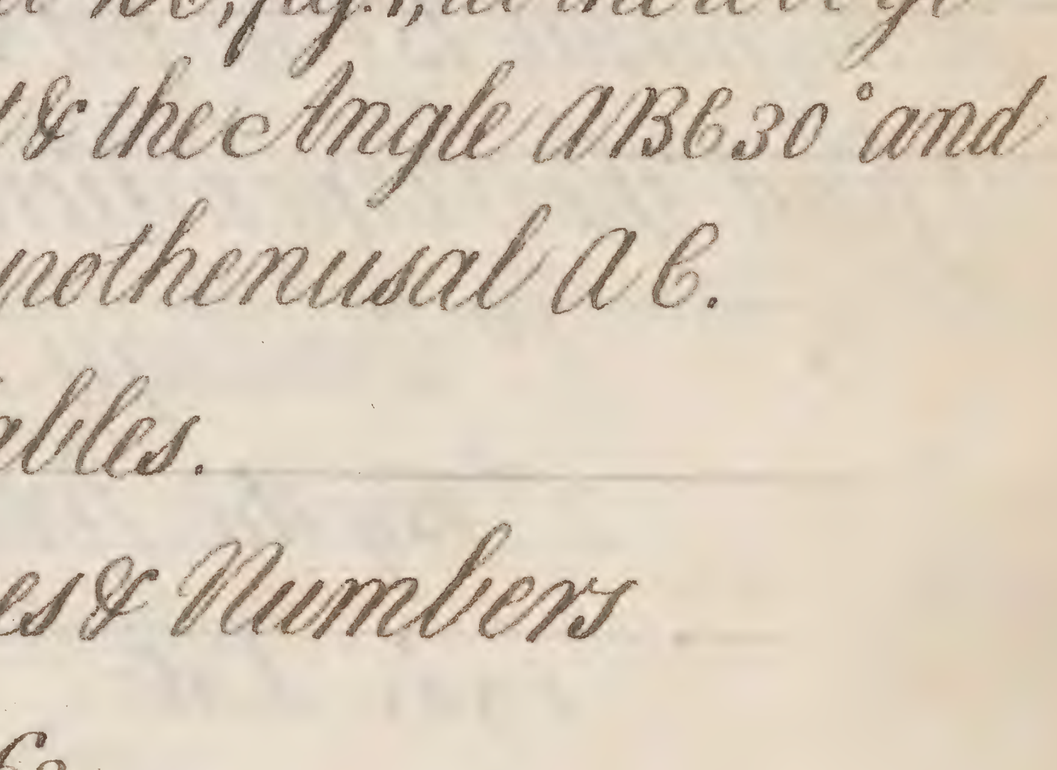
9.937531 : 2.602060 :: 10.0

10.

12.602060

9.937531.

462 = 2.664529



The compasses extended from 60° to 90° on the Sines reach from 400 to 462 on the Numbers.

Case 3.

The Hypothenusal & Angle at the Base being given to find the Perpendicular.

In the same Triangle (fig: 1) let there be given the Hypothenusal B6462 feet & the Angle at the Base 6BA 30° to find the Perpendicular 6A.

1.st Analogy by the Tables.

2.nd By the Lines of Sines & Numbers.

s. 90° : L. 462 :: s. 30° : L. 231

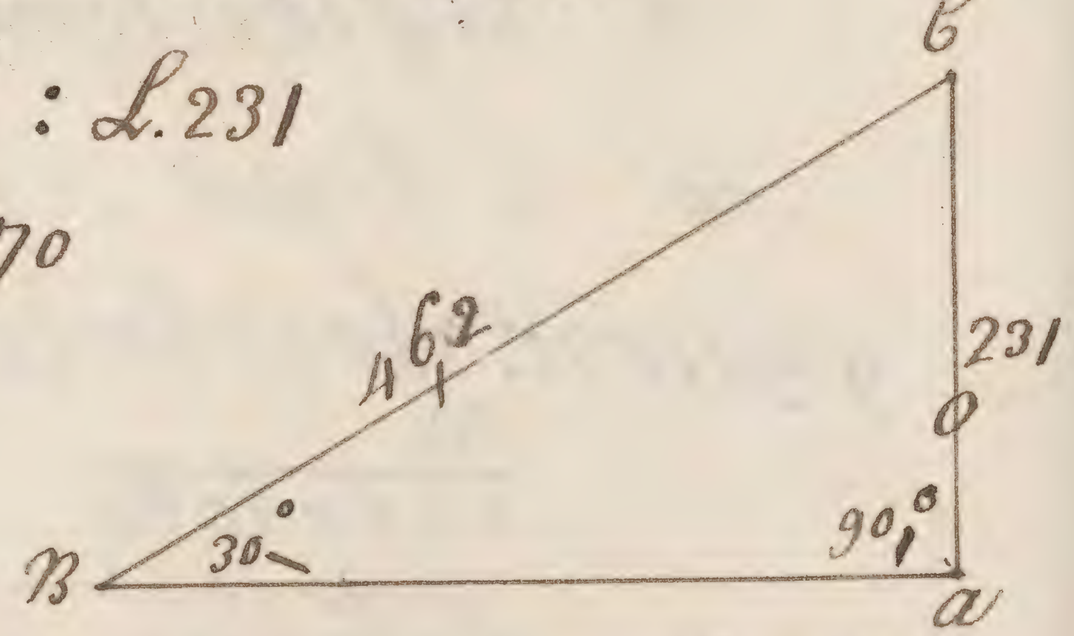
10 : 2.664642 :: 9.698970

9.698970

12.363612

10

231 = 2.363612



The compasses extended from 90° to 30° on the Sines reach from 462 to 231 on the Numbers.

Case 4.

The Hypothenusal & Angle at the ~~Base~~ being given to find the Base.

Let there be given in the former Triangle (fig: 1) the Hypothenusal B 462 feet & the Angle at the Base CBA 30° then By consequence the Angle BCA must be 60° the complement of the other to 90°. Quære to find the Side BA.

1.st Analogy by the Tables.

2.nd By the Lines of Sines & Numbers.

s. 90° : L. 462 :: s. 60° : L. 400

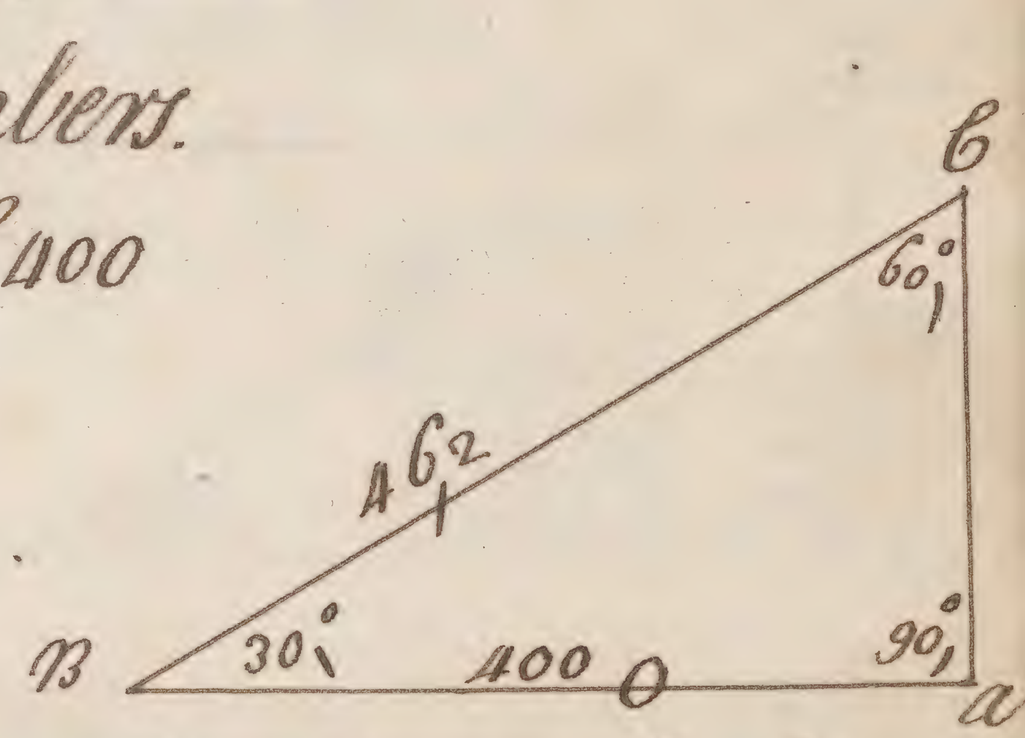
10 : 2664642 : 9.937531

9.937531

12.602173

10

400 = 2.602173



The Compasses extended from 90° to 60° on the Sines reach from 462 to 400 on the Numbers.

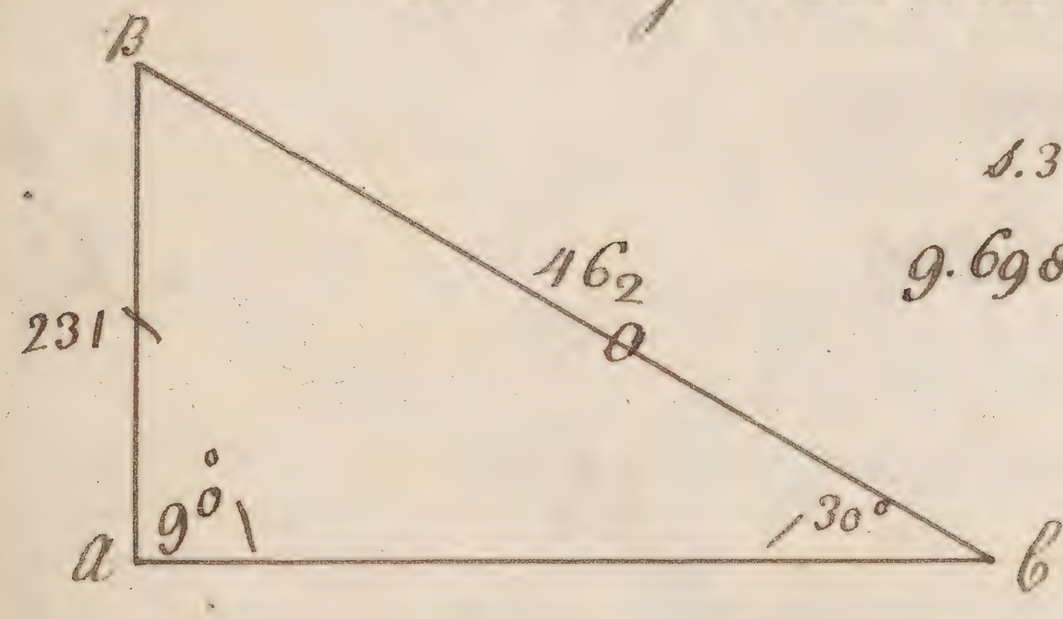
Case 5.

The Perpendicular & the Angle at the Base being given to find the Hypothenusal.

Ex. If the perpendicular 6A (fig:1) be given 231 & the Angle at the Basic 6BA 30.° the Hypothenusal B6 is requir'd.

1.st Analogy by the Tables.

2.nd By the Lines of Sines & Numbers.



S.30° : L.231 :: S.90° : L.462

9.698970 : 2.363612 :: 10

10

12.363612

9.698970

462 = 2.664642

The compasses extended from 30° to 90.° on the Sines reach from 231 to 462 on the Numbers.

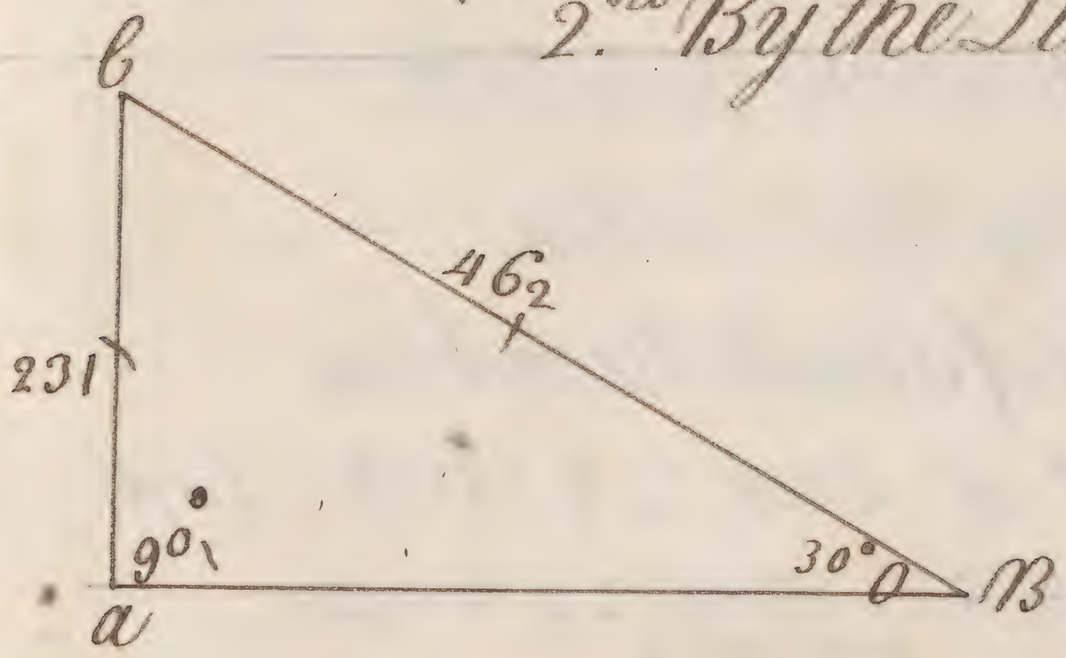
Case 6.

The Hypothenusal & Perpendicular being given to find the Angle at the Base.

In the same Triangle (fig: 1) there is given the Hypothenusal Bc 462 feet & the Perpendicular cA 231 feet & it is requir'd to find the Angle cBA.

1.st Analogy by the Tables.

2.nd By the Lines of Lines & Numbers.



L. 462 : s. 90° :: L. 231 : s. 30°

2.664642 : 10. :: 2.363612

10

12.363612

2.664642

30° = 9.698970

The compasses extended from 462 to 231 on the Numbers reach from 90° to 30° on the Lines.

Of The Obliquangled plain Triangles.

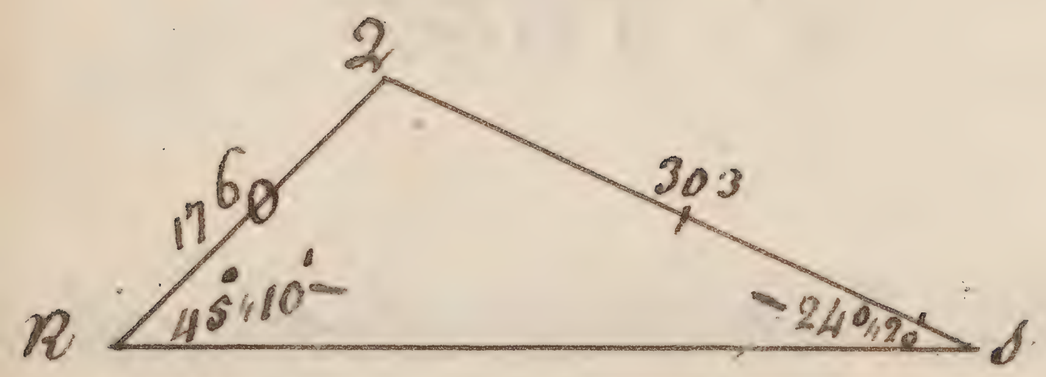
Case 7.

Having two Angles & a Side opposite to one of them given, to find the Side opposite to the other.

In the Triangle 2RS (fig: 4) there is given the Angle 2SR 24° 20', & the Angle 2RS 45° 10', & the Side 2S 303 feet & it is required to find the Side 2R.

1.st Analogy by the Tables.

2.nd By the Lines of Sines & Numbers.



S. 45° 10' : L. 303 :: S. 24° 20' : L. 176

9.850745 : 2.481443 :: 9.614944

9.614944

12.096387

9.850745

176 = 2.245642

The Compasses extended from 45° 10' on the Lines to 24° 20', reach from 303 to 176 on the Numbers.

In like Manner if the angle RDS 110° 30', & the angle 2RS 45° 10', & the Side 2S 303 feet had been given the Side RS required; you work as before.

1.st Analogy by the Tables.

2.nd By the Lines of Sines & Numbers.

S. 45° 10' : L. 303 :: S. 69° 30' : L 400

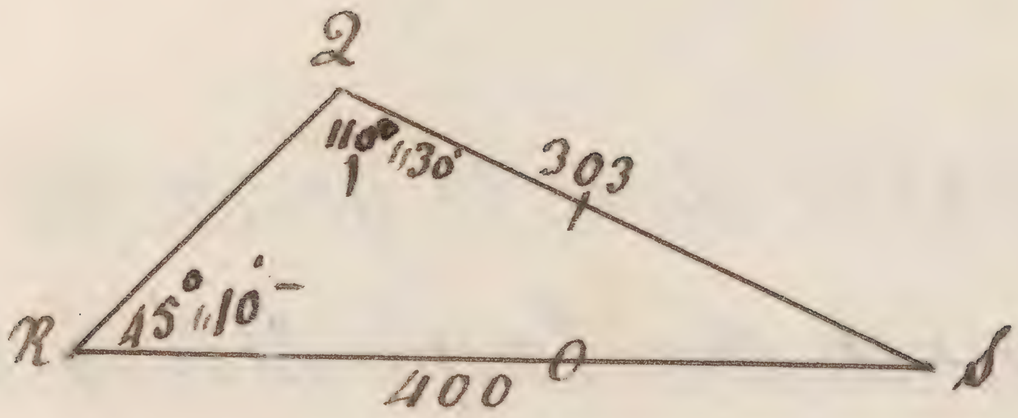
9.850745 : 2.481443 :: 9.971588

2.481443

12.453031

9.850745

400 = 2.602286



The compasses extended from 69° 30' to 45° 10' on the Sines, reach from 303 to 400 on the Numbers.

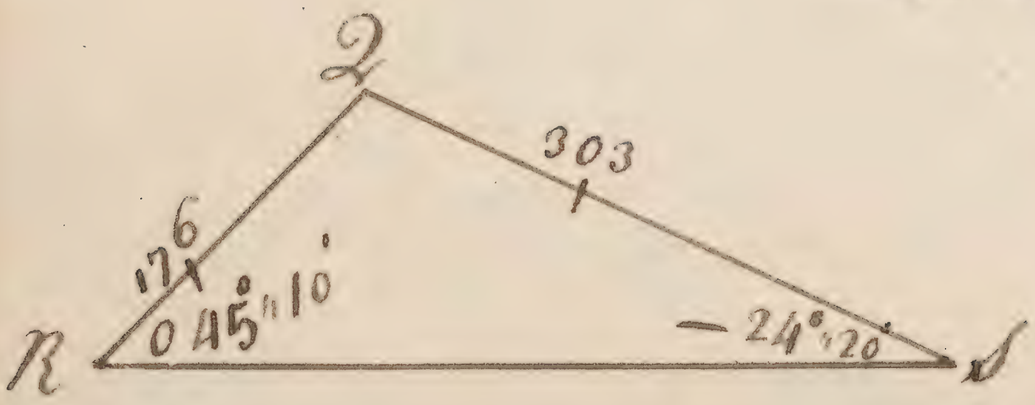
Case 8.

Two Sides & an Angle opposite toone of them given to find the Angle opposite to the other.

In the same Triangle (fig: 4) let there be given the Side 2S 303 & 2R 176 together with the Angle 2SR 24° 20' to find the Angle 2RS.

1st Analogy by the Tables.

2nd By the Lines of Sines & Numbers.



L. 176 : s. 24° 20' :: L. 303 : s. 45° 10'

2.245513 : 9.614944 :: 24814 43

9·614944

12·096387

2.245513

45° 10' = 9·850874

The Compasses extended from 176 to 303 on the Numb.rs reach from 24° 20' to 45° 10' on the Sines.

When the side opposite to the sought Angle is less than the Side opposite to the given one, the sought Angle is doubtful & may be either that found in the Tables, or its Supplement to a Semicircle; both of them being possible.

Case 9.

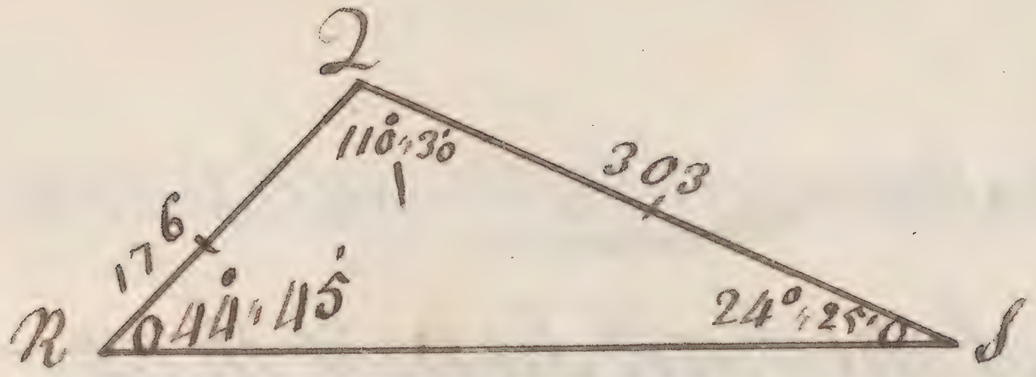
Having two Sides & the Angle contained by them given to find either of the other Angles.

Suppose there were given the Side 2 S 303 (fig: 4) & the Side R 2 176 & the Angle comprehended by them R 2 S 110° 30', to find either of the other Angles.

As the half Sum of the comprehending Sides,
Is to their half Difference.
So is the Tangent of the half Sum of their opposite Angles.
To the Tangent of their half Difference.
Which added to their half Sum gives the greater Angle
And subtracted from their half Sum gives the lesser Angle
requir'd.

1.st Analogy by the Tables.

2.nd By the Lines of Tangents & Numbers.



176	303	180° 0'
303	176	110°30'
2 | 479	2 | 127	2 | 69°30'

L. 239.5 : L. 63.5 :: t. 34°45' : t. 10°20'

2.378398 : 1.802774 :: 9.83984

9.83984

11.642614

2.378398

10°20' = 9.264216

34°45'	34°45'
10°20'	10°20'
44°55'	24°25'

The Compasses extended from 239.5 to 63.5 on the Numbers reach from 34°45' to 10°20' on the Tangents.

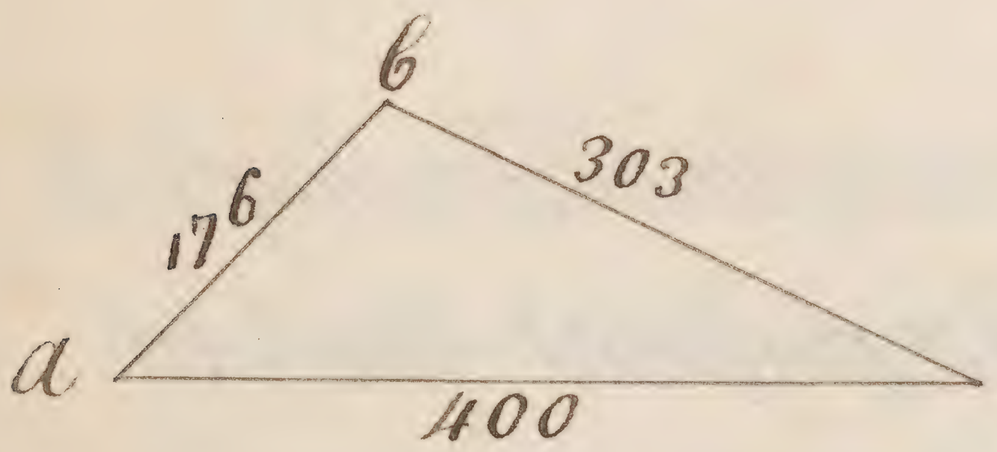
Case 10.

The three sides of a right-lined plain Triangle being given how to find the Area, or superficial Content thereof.

Rule.

From the half Sum of the Sides subtract each particular Side; then the Logarithms of the half Sum & three Differences being added together, half their Sum will be the Logarithm of the Area required.

Let the Triangle given be a B C (fig: 4) the Sides thereof being 400, 303, 176 how much is the Area, or superficial content thereof?



400
176
303

2 | 879
439.5

439.5	439.5	439.5
400	176	303
39.5	263.5	136.5

439 = 2.642465
39.5 = 1.596597
263 = 2.419956
136 = 2,133539

2 | 8.792557

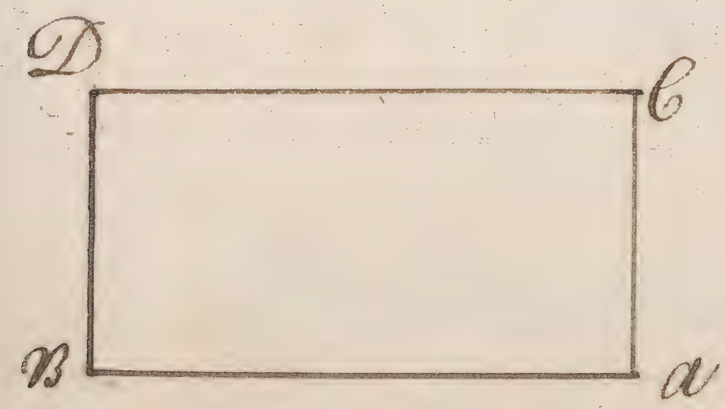
~~249048~~ = 4.396278 = Area

The Table of Logarithms or Line of Numbers expedite the casting up of the content of any Superficies & for Land-Measuring particularly, whose conciseness will appear by the solutions of the following Problems.

Problem 1.

The length & breadth of a right angled parallelogram, being given in perches to find the content thereof in perches.

In the parallelogram a B C D let the length a B be 25 perches & the breadth a C 12.8 perches to find the content



L. 1 : L 12.5 :: L. 25 : L. 320

0.000000 : 1.107210 : : 1.397940

1.107210

320 = 2.505150 = Area

Analogy { As 1 is to the Breadth in Perches,
So is the Length in Perches,
To the Content in Perches.

1.st By the Logarithms.

2.nd By the Line of Numbers.

The Compasses extended from 1 to 12.5, reach from 25 to 320 on the Line of Numbers.

Problem 2.

The Length & Breadth of a Parallelogram being given in Perches to find the Content in Acres.

So in the same Parallelogram AB being 25 Perches & A 6 12.8 Perches, the Analogy is

{ As 160 is to the Breadth in Perches
{ So is the Length in Perches
{ To the Content in Acres.

1.st By the Logarithms.

2.nd By the Line of Numbers.

L. 160 : L 12.8 : : L 25 : L. 2

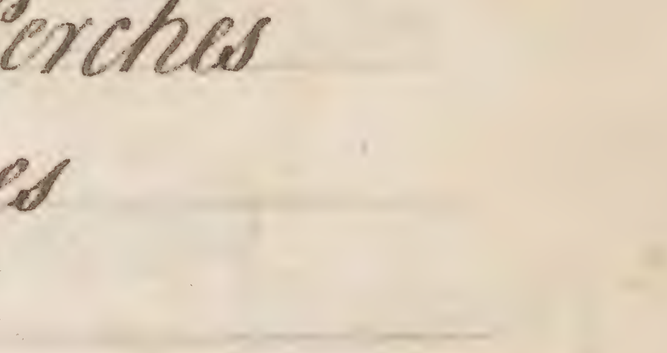
2.204120 : 1.107210 : : 1.397940

1.107210

2.505150

2.204120

Acres 2 = 0.301030



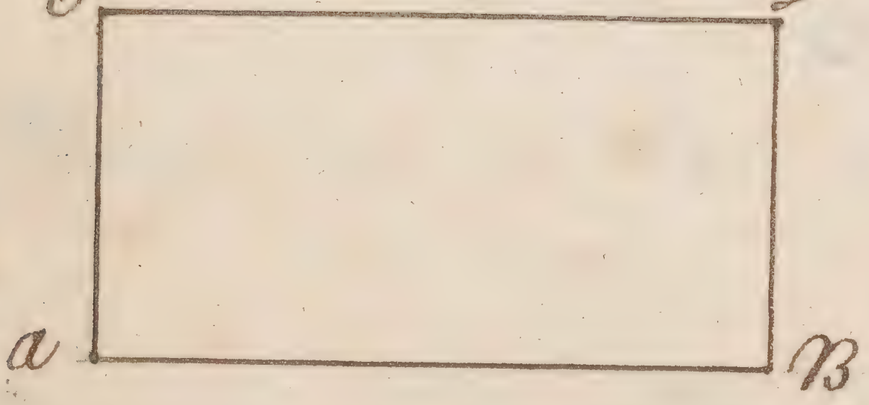
The compasses extended from 160 to 12.8, reach from 25 to 2 on the Line of Numbers.

Problem 3.

The length & breadth of a Parallelogram being given in Chains to find the Content in Acres.

In the Parallelogram A B C D let the Length thereof C D be 6 Chains 25 Links & the Breadth thereof B D 3 Chains 20 Links, to find the Content in Acres.

Analogy { As 10 is to the Breadth.
So is the length to the Content in Acres.



L. 10 : L. 3.20 :: L 6.25 : L. 2

1.000000 : 0.505150 : : 0.795880

0.795880
1.301030
1.000000
0.301030

2 = 0.301030 Acres

The Compasses extended from 10 to 3.20 reach from 6.25 to 2 on the Line of Numbers.

Problem. 4.

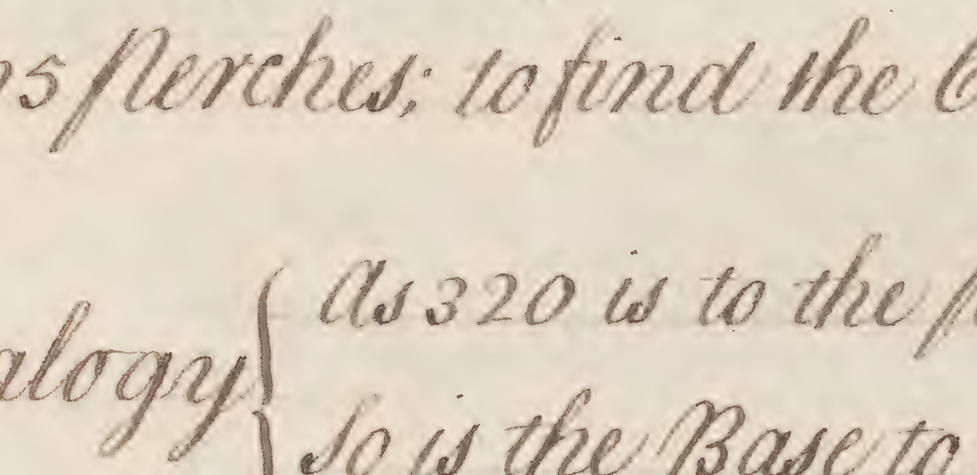
The Base & Perpendicular of a Triangle being given in Perches, to find the Content in Acres.

In the Triangle CED let fall the Perpendicular EF which suppose 38 Perches & 4 tenth parts of a Perch, & the Base CD 75 Perches: to find the Content in Acres?

Analogy { As 320 is to the Perpendicular.
So is the Base to the Content in Acres.

1st By the Loganithms.

2nd By the Line of Numbers.



L. 320 : L. 38.4 : : L. 75 : L. 9

2.505150 : 1.584331 : : 1.875061

1.875061
3.459392
2.505150

9 = 0.954242 Acres

The Compasses extended from 320 to 38.4 reach from 75 to 9 on the line of Numbers.

Problem 5.

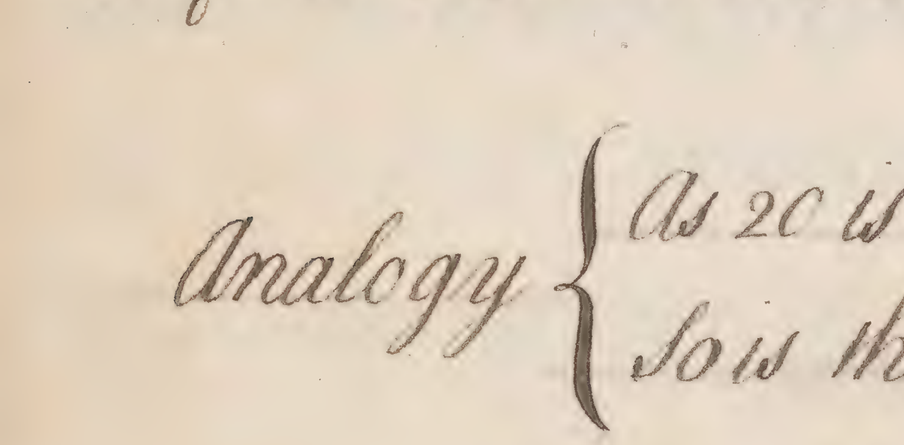
The Base & Perpendicular of a Triangle being given in Chains, to find the Content thereof in Acres.

So in the former Triangle, the Perpendicular E F being 9 Chains 60 Links & the Base C D 18 Chains 75 Links to find the Content in Acres?

Analogy { As 20 is to the Perpendicular.
So is the Base to the Content

1.st By the Logarithms.

2.nd By the Line of Numbers.



L. 20 : L. 9. 60 :: L. 18. 75

1.301030 : 0.982271 :: 1.271842

0.982271

2.254113

1.301030

9 = 0.953083 Acres

The Compasses extended from 20 to 9.60 reach from 18.75 to 9 on the Line of Numbers.

Problem 6.

Having the Length of a Furlong given in Perches to find the breadth of the Acre.

So if a furlong should be 50 Perches long & you would know what Breadth it must be to make an acre, this is the Rule.

As the Length of the furlong in Perches,
Is to the Constant Number 160.
So is 1 Acre,
To the Breadth of the Acre in Perches.

1.st By the Logarithms.

2.nd By the Line of Numbers.

L. 50 : L. 160 :: L 1 : L. 3.5

1.698970 : 2.204120 :: 0.000000
1.698970

Perches 3. 2 = 0.505150

The Compasses extended from 50 to 160 reach from 1 to 3.2 on the Line of Numbers.

Problem 7.

The Length of a Furlong being given in Chains, to find Breadth of the Acre in Chain Measure.

So if the furlong be 6 Chains 56 Links, how broad must it be to make an Acre? To perform which this is the Rule.

As the Length of the furlong in Links,
Is to the constant Number 100000.
So is 1 Acre,
To the Breadth of the Acre in Chain-Measure.

1.st By the Logarithms.
2.nd By the Line of Numbers.

L. 6.56 : L 100000 :: L. 1
2.816904 : 5.000000 : : 0.000000
152.44 fere = 2.816904 / 2.183096

The Compasses extended from 656 to 100000 reach from 1 to 152.44 on the Numbers.

Problem, 8.

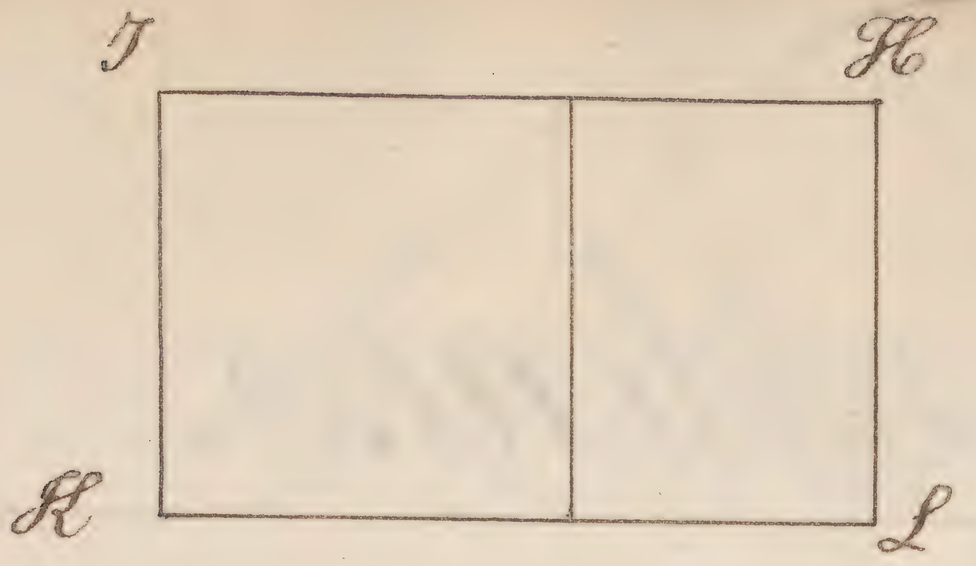
The Length or breadth of any piece of Ground being given either in Perches, or Chains & Links, how to lay out any Quantity of Acres required, retaining still the same Breadth or Length.

Suppose H I K L were a piece of Land the side whereof from H to I were 9 Chains 75 Links, & from that Side I would lay out a Parcel which should contain 7 acres; How much Breadth must be set off from H & L towards I and K? To perform this, this is the Proportion.

{As the Length or breadth given, is to 100000 (or 1 Acre)
{So is the Number of Acres requir'd to be laid out, to the breadth.

1.st By the Logarithms.

2.nd By the line of Numbers.



L.975 : L.100000 :: L.7

2.989005 : 5.000000 :: 0.845098

0.845098

5.845098

2.989005

718 = 2.856093

The compasses extended from 975 to 100000, reach from 7 to 718 on the Line of Numbers.

Hereof expect more in the laying out of Lands.

Problem 9.

The Area or superficial Content of any Piece of Land being given according to one kind of Perch to find how much the same Piece of Land would contain if it were measur'd with a Pole or Perch of another length differing from the former.

Suppose B be a piece of Land (suppose a Wood) which being measured by a Chain of 18 feet (which is the Woodland Perch) should contain 613 Acres; and it were required to find how many

Acres the same Land would contain if it had been measured by a Chain of 16½ feet to the Perch (which is a Statute Perch) for several Countries have their respective Customs; but this Rule is general: To work it, this is the Proportion.

As the Square of the Perch by which the Land is to be measur'd,
Is to the Square of the Perch by which it was measured:
So is the Area given,
To the Area required.

For similar Plains are in Proportion one to another as are the Quadrats of their homologal Sides.

1.st By the Logarithms.
2.nd By the Line of Numbers.

C

L. 16.5 = 1.217484
1.217484
2.434968 :

L. 18 = 1.255272
1.255272
2.510544 :: 1.787460
1.787460
4.298004
2.434968
Acres 73 = 1.863036

L. 61.3

The Compasses extended from 272.25 the Square of 16.5 to 324 the Square of 18, reach from 61.3 to 73, on the Line of Numbers.

On the contrary, If this piece being measured by a chain of 16½ in the perch should have contain'd 73 Acres, & it had been required to know how many acres it would have contained had it been measured by the Chain of 18 feet in a perch, the Operation as before.

L. 18 = 1.255272
1.255272
2.510544 :

L. 16.5 = 1.217484
1.217484 L. 73
2.434968 :: 1.863323
1.863323
4.298291
2.510544
Acres 61.3 = 1.787747

The Compasses extended from 324 to 272.25 reach from 73 to 61.3 on the Line of Numbers.

Problem 10.

The Area or Content of any Plat of Land being given (& the Scale lost or concealed) to find the Scale by which the same Plat was plotted.

Let the Figure B (in the last Problem) be a piece of Land said to contain 8 Acres & you desire to know by what Scale it was plotted.

First, take any Scale & cast up the content of the Plat thereby & so doing suppose you find the same Plat to contain 11.50 Acres which should by the Aſsertion be 8 Acres: Now to find the true Scale by which it was plotted, this is the Proportion,

as the Quantity of Acres found.
Is to the Square of the Scale by which you measur'd.
So is the Content given.
To the Square of the Scale by which it was plotted.

1.st *By the Logarithms.*

2.nd *By the Line of Numbers.*

$L.11.5 : L\frac{30}{900} :: L.8$

2.060698 : 2.954243 :: 0.903090

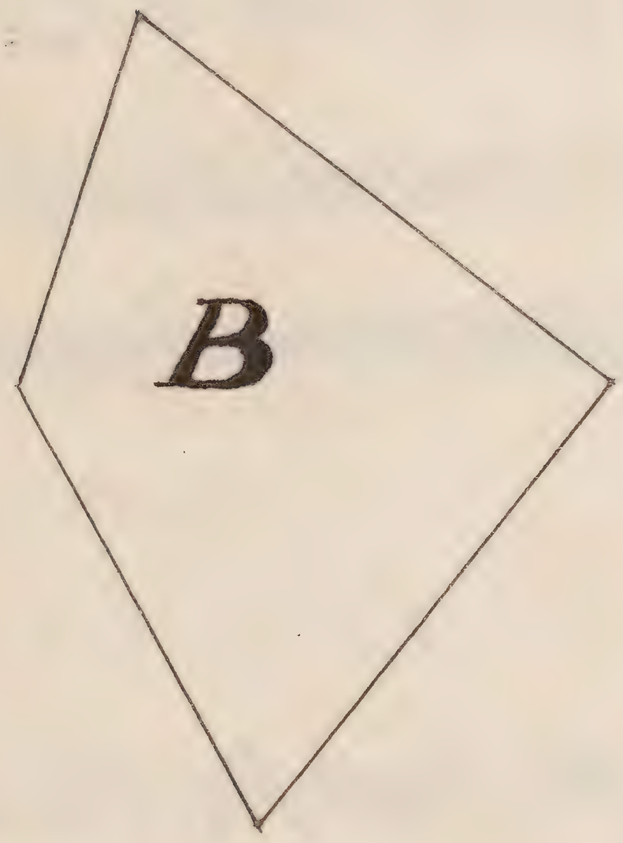
2.954243

3.857333

2.060698

2 | 1.796635

$8 = 0.8983175$



The Compasses extended from 11.5 to 900 reach from 8 to 62.6 on the Line of Numbers.

How to find how many Acres Roods & Perches are contained in any piece of Land the plot thereof being first taken by any Instrument.

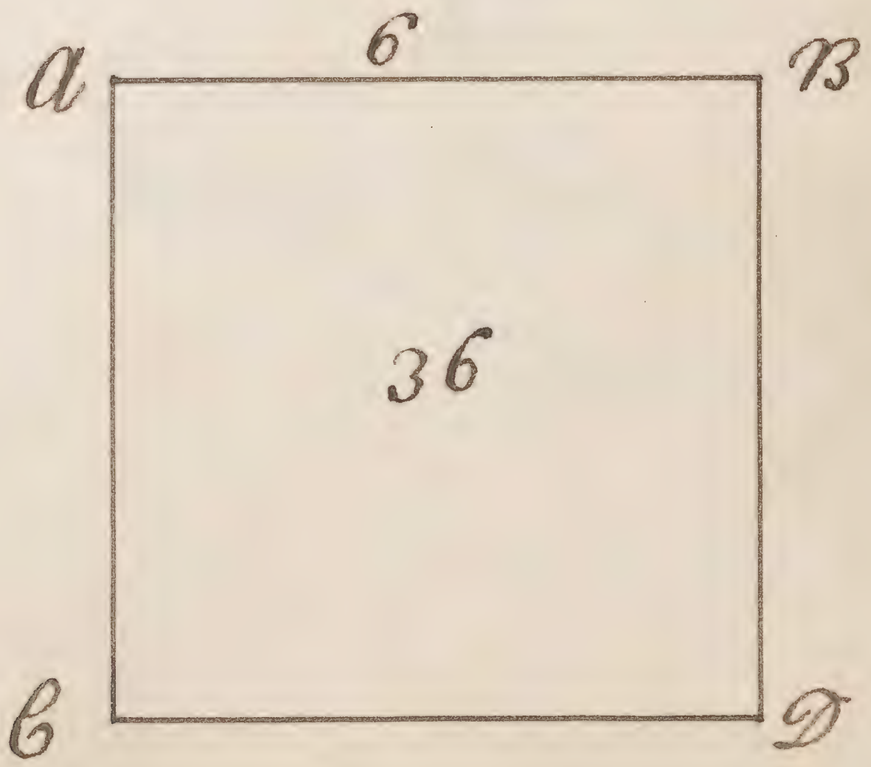
1.st of the Geometrical Square.

Suppose the a B C D to be a piece of Land & the Side thereof to contain 6 Perches to find the Content.

The Rule

Multiply one of the Sides into it self & the Product of that Multiplication shall be the Content of the Square.

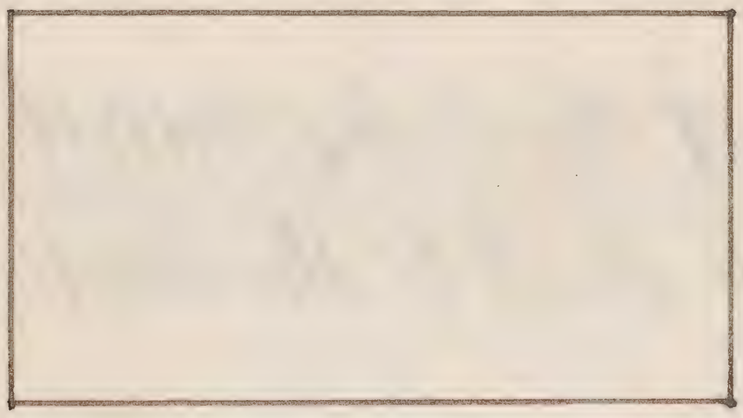
6
6
36



Of the long Square or Parallelogram.

To find the superficial Content thereof multiply one of the longer Sides by one of the Shorter.

Suppose the longer side of the Parallelogram to be 13 Perches, & the shorter 7 Perches. What is the Content?

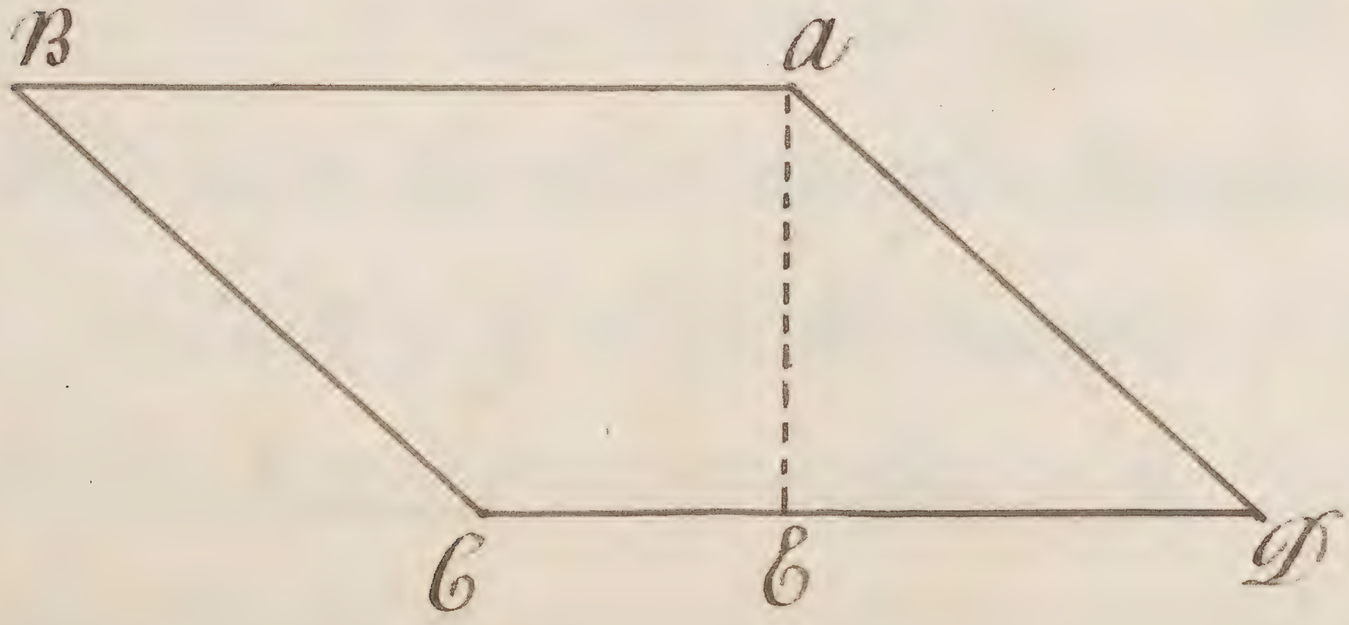


13
7
91 = Content

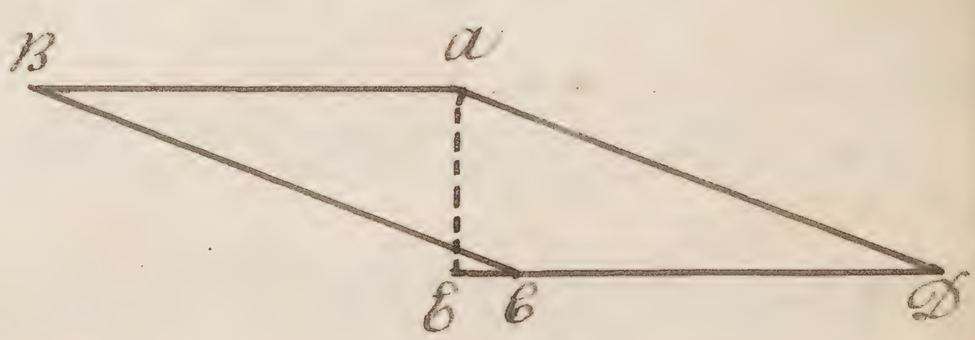
There are two other 4 sided Figures which may be said to have some Regularity & often fall out in Buildings Viz. a Parallelogram (as ABCD) having it's opposite sides both ways parallel.

To find the Content thereof multiply one of the sides A B by A E the perpendicular Distance of A B from it's opposite side C D (produced if need require it) & the Product is the superficial Content.

Suppose the shorter side of this Parallelogram be 9 feet, & the longer 11 feet, what is the Content?

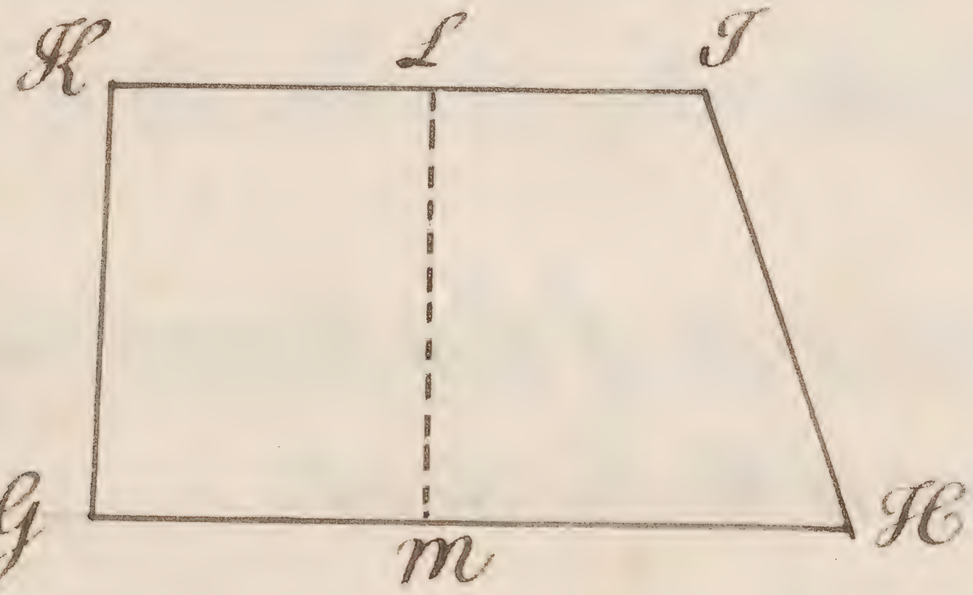


11
6
66



To find the content of the other 4 sided Figure or parallelogram, multuply half the Sum of the parallel sides G H & I K by their distance L M & the Product is the Content.

Suppose the shorter side of this parallelogram be 11 feet & the longer parallel side 14 feet. What is the Content?



14
11
2 | 25
12.5
8
100.0 = Content.

Suppose you were to find the Area or Content of the Triangle A B F the Base thereof A F containing 59 Perches and the Perpendicular E B 27 Perches.

Rules

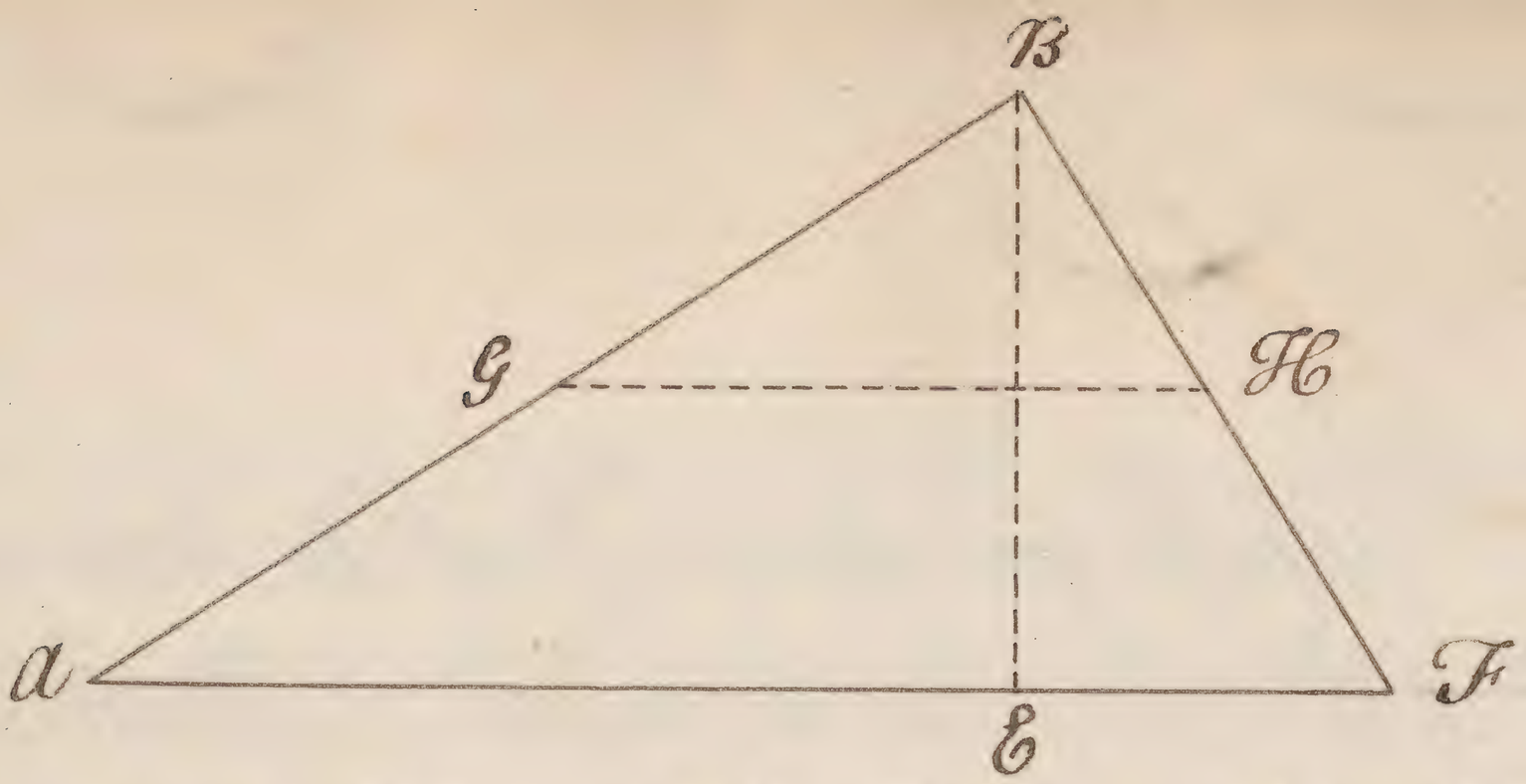
1.st Half the length of the Base being multiplied by the length of the Perpendicular shall be equal to the Area of the Triangle.

2.nd Half the Length of the perpendicular being multiplied by the whole Base will be the Content of the Triangle

3.rd The whole length of the Perpendicular being multiplied by the whole length of the Base half the product shall be the Content of the Triangle.

and of these three Methods, the fittest for Surveying is this last, the other two for Building.

Lastly if thro' the middle of the Perpendicular or sides be drawn a line parallel to the Base as G H then G H multiplied by the Perpendicular gives the Content; And this is common in measuring Buildings.



2 | 59 = aF
29.5
27 = BE
2065
590
1st 796.5

2 | 27 = BE
13.5
59 = aF
1215
675
2nd 796.5

59 = aF
27 = BE
413
118
2 | 1593
3rd 796.5 = Area

29.5 = GH
27 = BE
2065
590
Last 796.5 = area

Of the Trapezia.

The Trapezia consisting of four unequal Sides which is measured by first draning a Diagonal thereby reducing the figure into two Triangles then proceed as before to find the Area thereof.

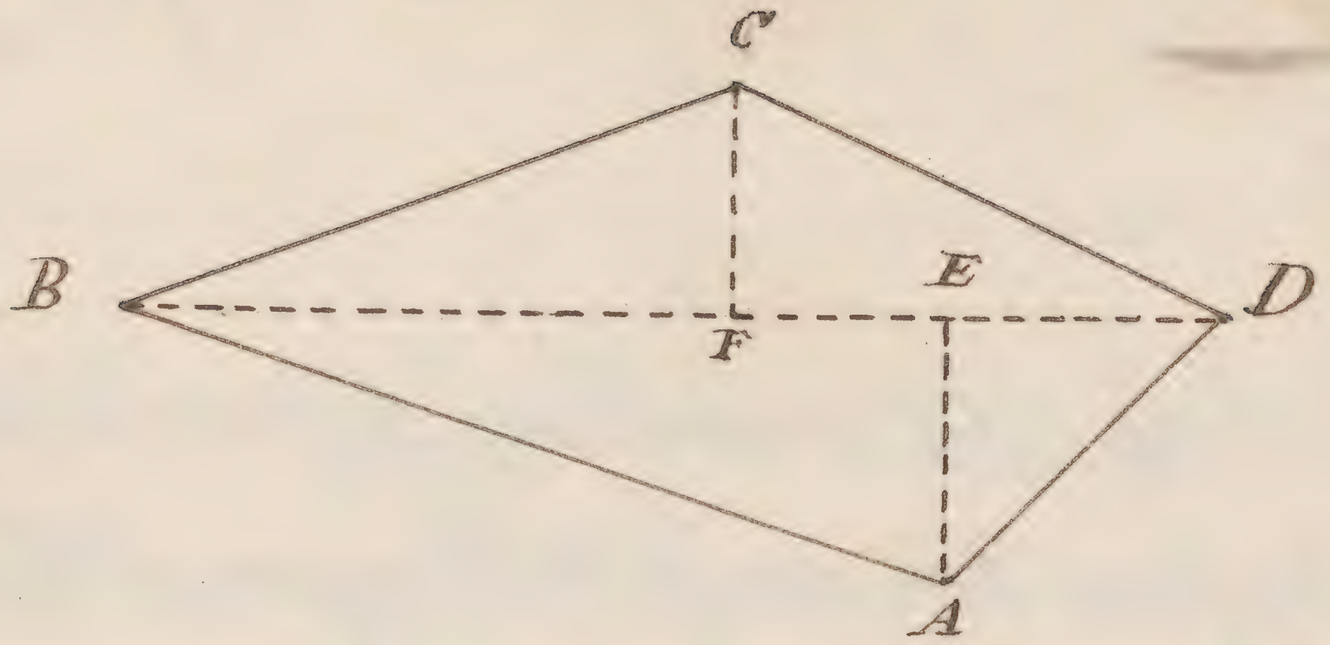
Suppose the Trapezia ABCD the Diagonal whereof BD 300 perches & the perpendicular CF 102 perches & the perpendicular AE 118 perches, Quære the area of the whole Figure.

as there are as many Varieties of measuring of Trapezia's as there are of triangles the Operation may be abbreviated thus.

The Base BD being commen to both the Triangles therefore add the two perpendiculars together, the half whereof being multiplied by the whole Base the Product will give the content of the whole Trapezia.

Or you may multiply the Sum of the perpendiculars by half the Base.

Or you may multiply the Sum of the perpendiculars by the Base & half the Product gives the Area of the Trapezia. This last Method is fittest for a Surveyor. The same Example as before.



102
118
2 | 220
110
300
33000 = area

102
118
220
150
33000 = area

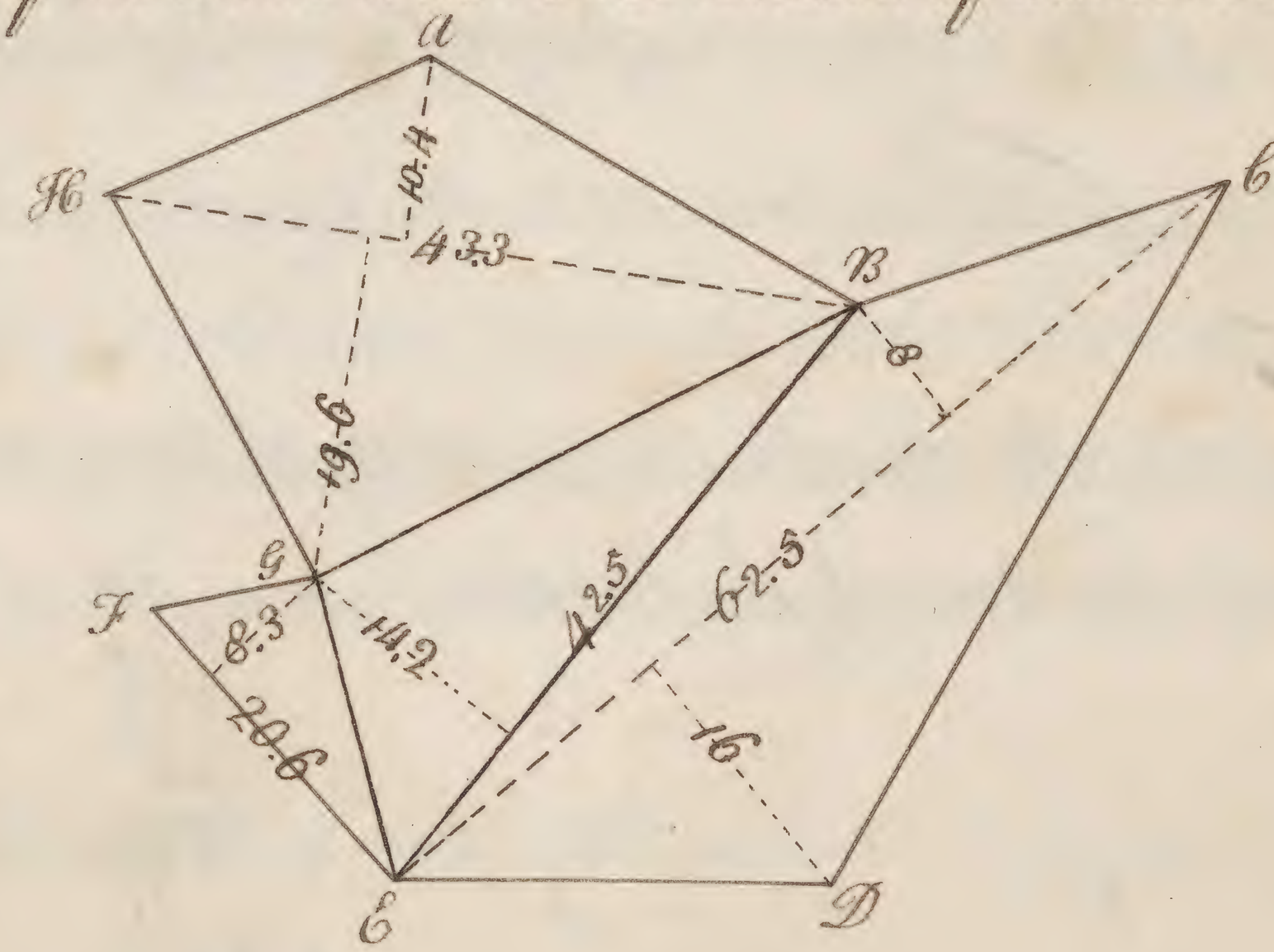
2 | 300
150

102
118
220
300
2 | 66000
33000 = area

Irregular Figures how to reduce them into Triangles or Trapezias & to cast up the content thereof.

Let A B C D E F G H be the figure of a field drawn either upon a Plain Table or otherwise protracted upon Paper. Before it can be measurd you must reduce it as before directed.

Or you may add the double area of all the Trapezias & Triangles together & then half the Sum will be the content of the whole.



62.5 × 24 = 2500
1250
150 0.0

43.3 × 30 = 12990

16 × 8 = 24

19.6 − 10.4 = 30.0

42.5 × 14.2 = 850
1700
425
603.50

20.6 × 8.3 = 618
1648
170.98

170.98
1299.00
1500.00
3573.48
1786.74 = Area

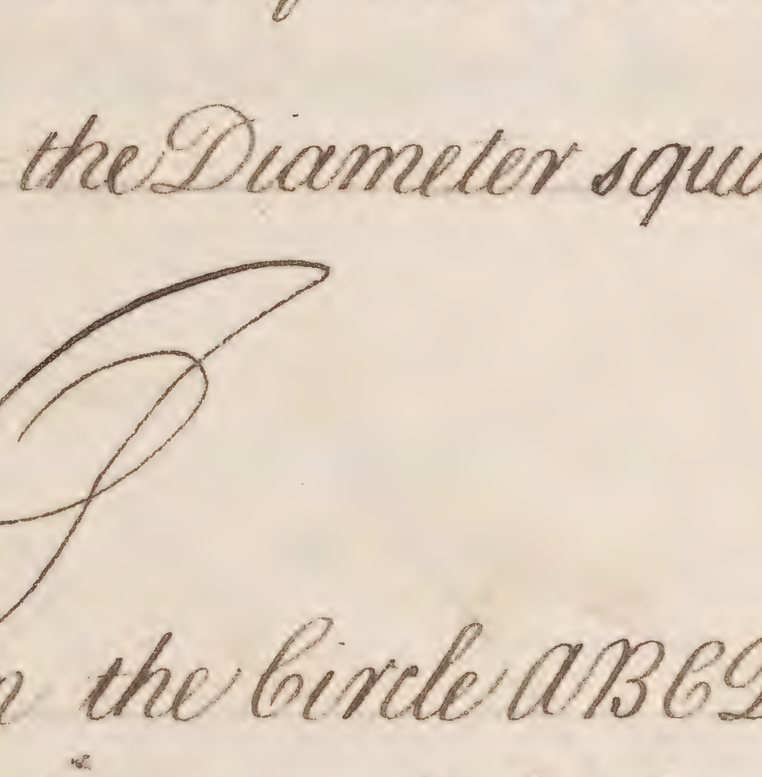
To find the Area or superficial content of a Circle.

Rule.

Multiply the Diameter thereof into it's self & that Square multiplied by 11 the Product divided by 14 quotes the area of the Circle.

Or ½ the Diameter squared & multiplied by 3 1/7 gives the Con:

In the circle ABCD let the Diameter thereof be DB 28 to find the Area of the Circle?



28
28
224
56
784
11
14 { 2 | 8624
     7 | 4312
616 = Area

14
14
196
3 1/7
588
28
616 = Area

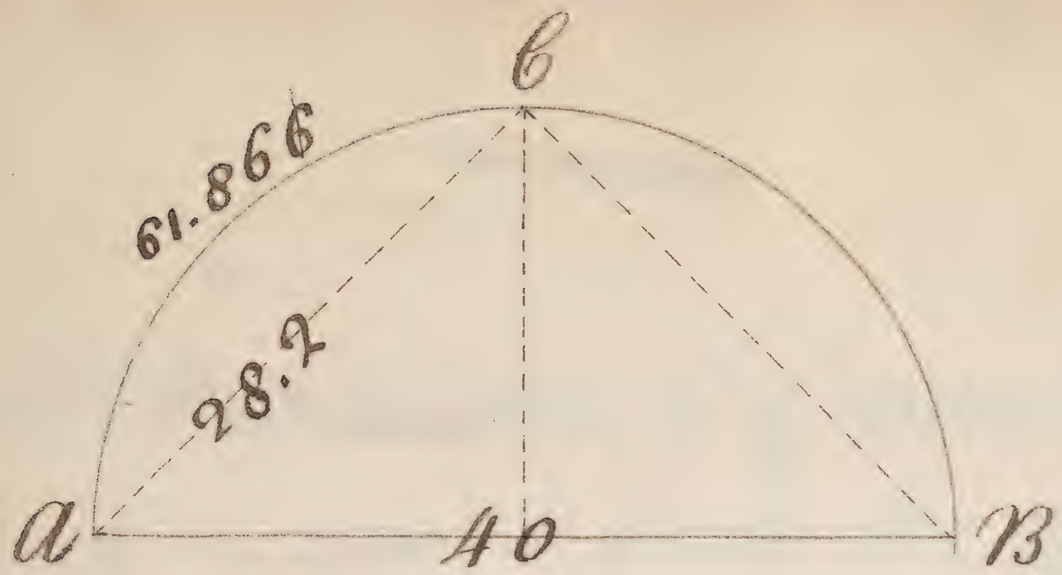
To find the Area or Content of a Semicircle, Quadrant, or other Sector of a Circle.

These portions or parts of Circles are measured by this one general Rule, Viz.

Half the Length of Arch line multiplied by the Radius, or Semidiameter gives the Content of that Semicircle, Quadrant, or Sector.—

To find the length of the Arch. Draw a Chord line from the two Extremes of the Arch which bisects perpendicularly up to the Arch & where that bysects the Arch draw a Chord line to either Extreme of the Arch. Then from 8 times that last Chord line take the first Chord line & ⅓ of the Remainder gives the approximated length of the Arch.

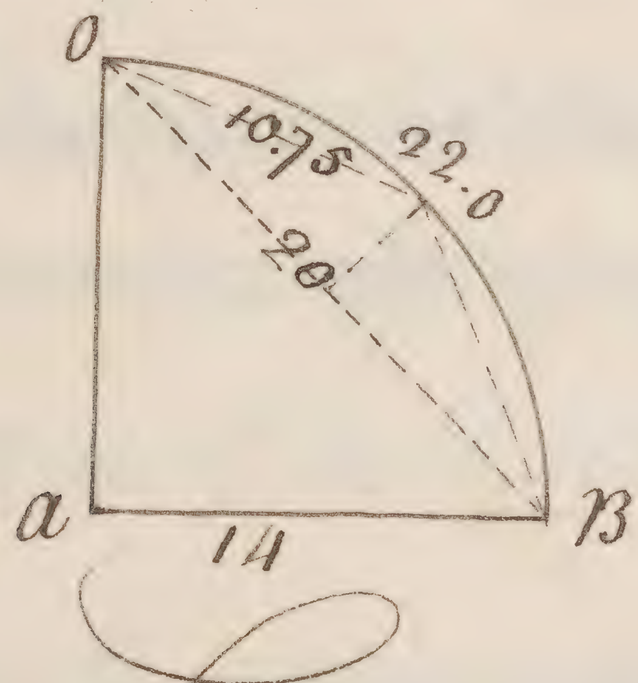
Let a Semicircle of any Extent A B be given to find the Content thereof?



28.2
8
225.6
40
3 | 185.6
61.86¢

2 | 61.86¢
30.93$
20
61866 = area

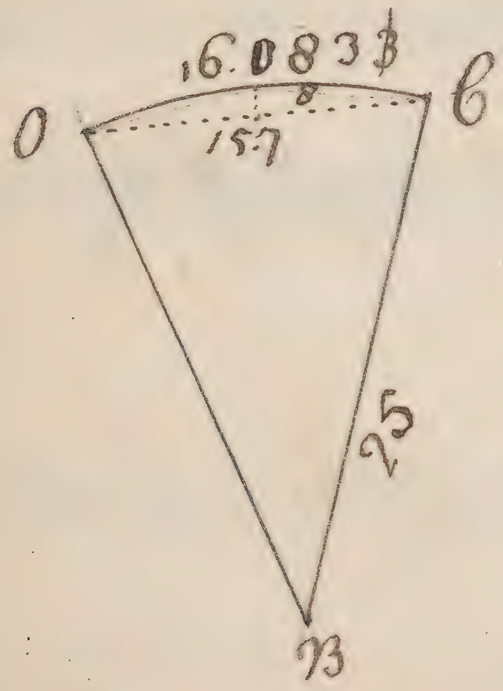
Let a Quadrant of any Extent O A B be given.
to find the Area?



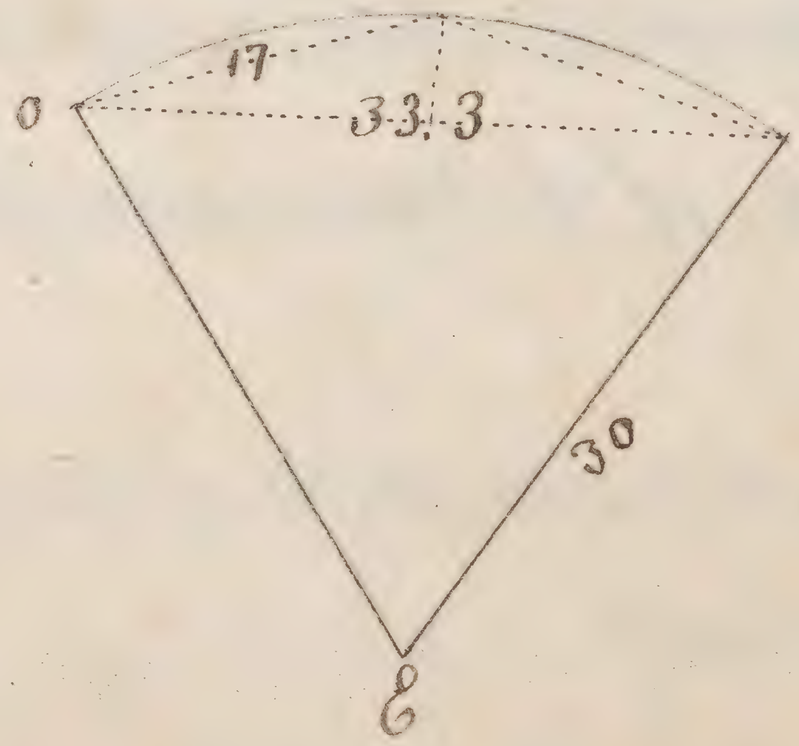
10.75
8
86.00
20
3 | 66
22

2 | 22
11
14
154 = Area

Let a Sector less than a Quadrant be given.
as OBC or OBC, to find its Area?



8
8
64
15.75
3 | 48.25
16.083$
8.04166
25
40 20830
160832
201.04150 = Area



17
8
136
33.3
3 | 102.7
2 | 34.2$
17.11¢
30
513.480 Area

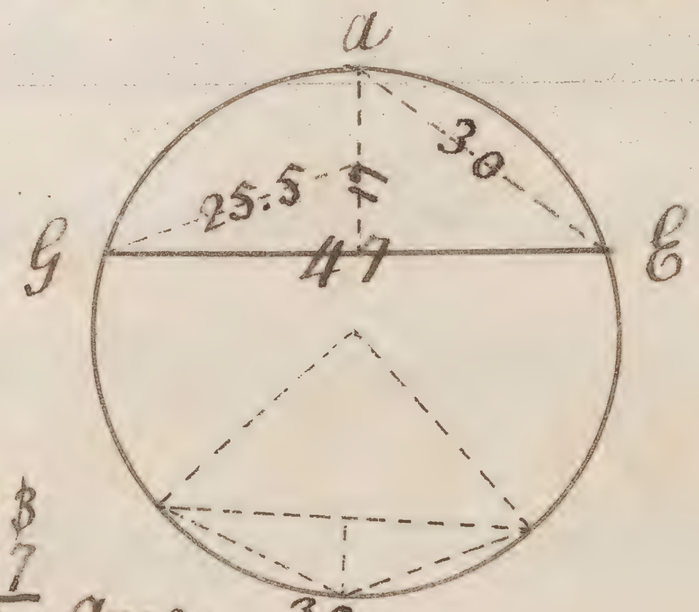
To find the Area of a Segment of a Circle.

The Solution of which Problem may be approximated thus.

The Chord-line drawn from the two Extremes of the Arch & perpendicularly bisected up to the Arch. $\frac{2}{3}$ of the first multiplied by the whole Length of the last gives the near Area of the Segment.

Nearer thus. Find the middle of the bisecting line from thence draw a line to one Extreme of the Arch then add the Chord-line extended from where the bisecting line cuts the Arch to one Extreme of the Arch, to four times that line drawn from the middle of the bisecting Line & $\frac{1}{3}$ of the Sum multiplied by $\frac{1}{5}$ of the quadrantal Arch gives the Content of the Segment as near as Archimedes's Proportion for the whole Circle.

Let it be required to find the Content or Area of the Segment a G E?



47
2
3 | 9 4
31.3 $\frac{1}{3}$
17
532.61 Area

Of Surveying by the Chain only.

A Surveyor if required to take a Distance, measure a Field, Close, or Wood, & to make a plot of the same upon Paper or Parchment may perform the same by a Chain only without any graduated Instrument by the following Examples

How to take an inaccessible Distance by the Chain only.

Let A be a Tree (or other Object) & you being at B should be required to tell how far the Tree at A is from you.

1.st Standing at B (which call your first Station) look towards A and cause somebody to move so (at some convenient Distance from B) that he may stand between your Eye & the Object at A as at D and there let him set up a Small Stick or Mark.

2.nd From B measure out any Number of Chains & Links to any Mark set up (either on the right or left hand from B) as to b & there set up another Mark (which call your second Station:) And

3 From thence look towards the Object at A: & at some convenient Distance from b in the visual Line bA cause one to set up a small Stick or Mark as at G.

4 In any two places of the line betwen B & b cause two Marks to be set up as one at E & the other at F.

5 Measure the Distances BD, BE, DE, & also the Distances bG, bF, GF: setting them down on a piece of Paper with their respective lengths to them.

From Station 1.st at B to Station 2.nd at b 52

From
- B to D —— 46.5
- B to E —— 17
- D to E —— 49

- b to F —— 18
- b to G —— 43.3
- F to G —— 40.8

How to protract or lay down the former Measures.
& thereby find the required Distances from B or b to the Object at A.

This is performed by the 8.th Geometrical Problem.

1. Upon Paper or Parchment draw a right Line at Pleasure as H K.
2. From your diagonal Scale take the Distance between your two Stations & set them upon the Line H K from B to b.
3. Take the Distance between B D or b E setting one foot of the compasses in B & describe the obscure Arch E a D.
4. From the same Scale take the Distance D E & set it upon the obscure Arch from E to D
5. Through the points B & D draw a right Line at pleasure as the line B D M, then
6. Take the distance b F or b G with which describe the obscure Arch F f G.
7. From the same Scale take the Distance F G which set upon the obscure Arch from F to G. and
8. Through the two points b & G draw a right Line at pleasure a b G g crossing the former line B D M in the point A which is the point where the Object standeth.

9. If you take the length of the Line B A & measure it upon the same Scale from whence you took your former Distances gives you the Distance of the Object at A from your first Station.

10. C A being measured by the same Scale gives you the Distance of the Object at A from your second Station.

from { Station 1.st at B to A – 139
Station 2.nd at C to A – 148

How (by a Chain only) to take the true Plot of an irregular Field, consisting of many Sides & Angles: & how to make a perfect draught of the same upon Paper or Parch-ment.

Let ABCDEFGHIKLMR be an irregular Field to be measured by the Chain only.

1. At your first Entrance into the field cause Beacons or other visible marks to be set up at (or as near as you can to) all the principal Angles or Corners thereof: as those at M, N, O, & P.
2. Consider the fittest or most convenient Corner to begin at (al-tho' any will serve) as is here done with the Beacon or Mark at O.
3. Then laying one End of the Chain to the Beacon at O, measure out 2 Chains (more or less as you see Occasion) from the Beacon at O towards the Beacon at P & at the end of 2 Chains (or other Mea-sure) stick up a small mark or Stick. Again from your Bea-con at O measure out $2.\overset{6}{45}$ (or other Measure) from the Beacon

at O towards the beacon at N& there set up another small stick or Mark: And then Measure with your Chain the Distance between the two Sticks or Marks which suppose to be 6 2.91.

4. Prepare a Book or Sheet of Paper ruled as is here done to set down your Measures as you go about the Field: But first at the top of the Book or Paper make a Triangle answerable to that which you measured out upon the Ground in the Field & set such Numbers, Letters, & Marks to it as you there measured & designed to know them by.

Sets off	6h: L.	Sets off.
0. 30	0. 00	
0: 70	1. 40	
0. 50	3. 40	
1. 20	6. 50	
0. 40	7. 20	
0. 70	0. 00	
touch	2. 10	
0. 90	3. 35	
0. 40	4. 80	
1. 10	6. 50	
0. 25	0. 00	
0. 50	1. 80	f 0. 70
	3. 00	a Con-
touch	4. 25	duit
0. 40	6. 50	Head
	0. 00	0. 35
	2. 70	0. 90
	5. 90	0. 30

This done begin your work at your Beacon at O & measure the distance of it from the Hedge on your left hand & finding it to be 30 links set them down on the left hand Column of your book against 6 0. 00 (or at the beginning of your Measure) because the Hedge was on your left Hand.

Then with your Chain measure from the Beacon at O towards the Beacon at N. in a right line: And as you go along at the end of 6 1. 40 you find a Break or Bend in the Hedge which is distant from your Chain-line 70 Links. Set down the 6 1. 40 in the middle Column & the 70 Links against it in the left hand Column of the Table. Then going on farther towards N. at the end of 6 3. 40

you come against another Break or Bow in the Hedge which is distant from your chain line 50 links both which set down in your Book or Paper in their respective Columns. And going on farther (in a straight line) towards P. at the end of 6.50 you find another Break or Bend which is distant from your Chain-line 1.20 both which set down in their respective Columns. And so measure on to your Beacon at P, which will terminate at the End of 7.20 this set down in the middle Column. And because the Beacon at P is 40 links distant from the Hedge set 40 Links in the left hand Column against 7.20 in the middle Column. And then draw a Line cross your Book or Paper to signify that you have done with that Side of the Field.

Then in your Book in the middle column under ch. Lin: write 0. 00. & because the Beacon at P is distant from the Hedge on your left hand 70 links set 70 links in the left hand column of the Table against 0.00; then go on & measure towards the Beacon at M & measuring on at the end of 2.10 you find that chain line just touches the corner or Bow of the Hedge; wherefore set down 2.10 in the middle column & against it in the left hand column write touch. And measuring on farther towards M at the end of 3.35 you find another Bow in the Hedge distant from the chain line 90 Links both which set down in their proper columns. And measuring on farther against 4.80 you find another Bow distant from the chain-line 40 links both

which set down. Then measuring on to your Beacon at M you find the Length to be 5.50 which set down in the middle column, and because the Beacon at M is distant from the Chain-line 1.10 set 1.10 in the Left Hand column against 5.50. And thus having finished this side of the Field you therefore draw a Line cross your Book as before.

In this manner must you proceed with the other two sides (& Chain-lines M N & N O) in the Field & set the several Distances at every Bend & the Distance of every bend from the Chain-line as you see is here done in this Table.

How to protract or lay down upon Paper or Vellom any irregular Piece of Ground measured by the Chain only as in the foregoing Work.

1st Draw a right line at Pleasure towards one end whereof assume a Point, as at O representing the place where you set up your first Beacon.

2. With your Compasses from any Scale of equal parts take out 2 Chains & set them upon the Line before drawn from O to x also out of the same Scale take 2 Chains 91 L. & setting one foot of the Com

passes in the point x with the other foot describe an obscure Arch of a circle Z Z: and then take 2 C. 15 L from the same Scale; & setting one foot of the Compasses in O. with the other describe an obscure Arch x y cutting the former Arch in the in the point I thro' which point dran the right line O I N at pleasure: And now you have upon your Plaper a Triangle in all respects like to that you measured out in the field & agreable also with that at the Top of your Book or Paper.

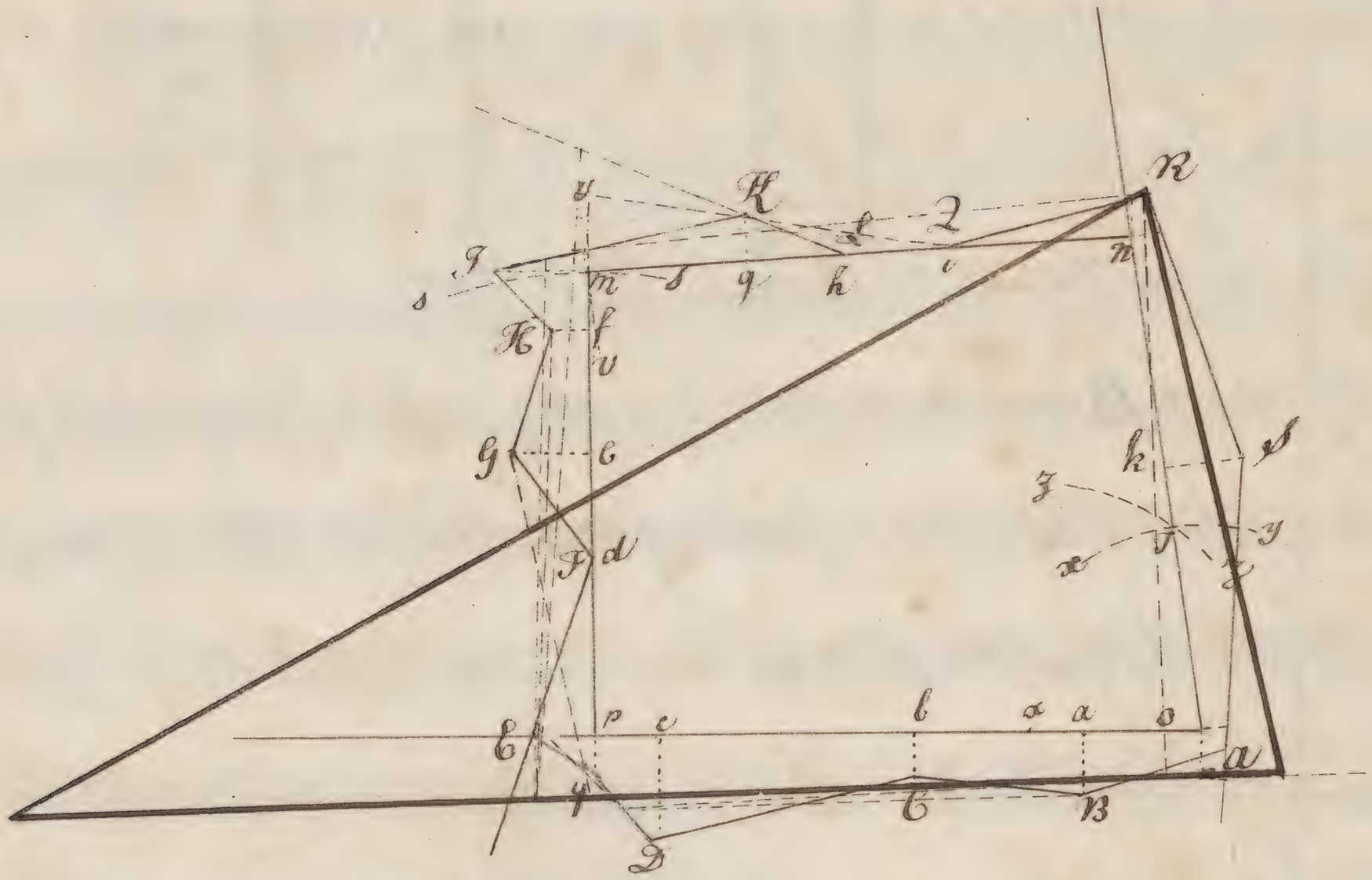
3. Lay your Book or Table before you by which you shall find that your first Chain line did contain 7620 L which take from your Scale & set them from O to N so is N the place of your second Beacon. Again by your book you find that the Distance between your first Beacon at O & your last at N was 5690 L which take from your Scale & set them upon the Line O I N from O to N so is N the place of your last Beacon. And now finding by your Book that your second Station for Chain-line doth contain 5 650 L take that length out of your scale & setting one foot of the Compasses in N with the other describe the obscure Arch S S. And the length of your third Chain-line being 6 650 L. take them from your Scale & setting one foot in N with the other foot describe the obscure Arch V V crossing the former Arch S S in the Point M which is the

point for your third Beacon: & thus have you drawn upon your Paper your four Chain lines making the quadrilateral figure MNOP.

4. Having gone thusfar you must have recourse to your Book again & there finding that at your Beginning at O your Bacon at O did stand 30 L. distant from the Hedge; take them from your Scale & set them from O to n. Also at the End of 16.40 L at at a Bon of the Hedge n was distant from the Chain Line 70 L. From your Scale take first 16.40 L. & set them from O to a & the 70 L from a to B & then draw the line n B extending it to A. Then take 36.40 L. & set them from O to b, & 50 L. from b to 6 & draw the Line B 6. Then take 66.50 L. & set them from O to c & 16.20 L. & set them from c to D & draw the Line 6 D. Then at 76.20 L. which is at the Beacon P set 40 Links to the Hedge at q & through that point draw the Line D q extending it to E & thus is this first side of your field finished.

5. Begin again at the Beacon at P where you find by your Book that the Bacon n was distant from the hedge 70 L. set 70 Links from P to 6 & draw the Line D q E. Then at 2 60 L. you find that your Chain Line did touch the Bon of the hedge: wherefore take 26.10 L. from your Scale & set them from P to F (or d) & draw the line E F. In like manner you find that at 36.35 L. the Bon of the hedge was distant from the chain line 90 L. Set the 36.35 L. from P to e & the 90 L from e to G & draw the line F G. Then your next length 46.80 L. will reach from P to f where the bend of the Hedge is distant from the Chain-line 40 L. Set the 46.80 L. from P to f & the 40 L. from f to H & draw the line G H. Then your last length 56.50 L. will reach

from ſl to M where the Beacon at M is distant from the hedge 16.10 L. set the 16.10 L. from M to I & draw the Line H I. Thus have you finiſhd your second Side & in the same Manner must you deal with the other two Sides, & in so doing you shall have the true Plot of the irregular piece of Ground upon your Paper which you must cast up in Acres Roods & Perches as before taught.



15.20
7.03
4560
106400
2 | 106.8560
10 | 53.428
Acres 5. 3428
4
Roods 1.3712
40
Perch: 14,8480

How to take the Plot of a Wood (into which you can-
not come to measure) by going round about the same by the
Chain only: And to make a Plot thereof upon Paper or Vellom.

Let Wbe such an irregular piece to be measured & plotted.

Go about the same & as near as you can conveniently to the Hedges
sides that inclose the Wood, & at all eminent turnings or corners thereof set
up Marks or Beacons as at A, B, C, D & E. then

1. At the Beacon B measure out $\frac{6}{2}$ (or any other Measure) from B to-
wards A & also towards C fixing up Staves also measure the Dis-
tance between those two Staves.

2. Also measure the Beacon at C, measure out towards the Beacons
at B & D $\frac{6}{2}$ (or other measur) as at the last Beacon B. And then

3. Having prepared a Field book ruled like this in the Margin.
Begin to measure as followeth, Viz.

1. From A towards B, where the Beacon at A stands 75 links
from the Hedge of the Wood wherefore against $\frac{6}{0}$ $\frac{L}{00}$ (in the

middle Column of your field Book) set 75 links in the left hand Column because the Hedge was on your left Hand. Then measuring on in a right line towards B where at 0.50 (6 L) your Chain line is distant from the Hedge 60 links set down 0.50 (6 L) in the middle Column & against it 60 Links in the left hand Column. Then

Measure on farther towards the Beacon at B you find that at

6 L	
1	05
1	75
3	10
4	80

your Chain line is distant from the Hedge

6 L
0.50
0.30
0.75
0.25

all which set down in your field Book as you see done in the Margin.

2. And now being arrived to your Beacon at B make in your field Book a Triangle like unto that which you set out in the Field at the Beacon B setting to it the same Measures as you there measured & then

3 Begin to measure from B (in a right line) towards & measuring along you will find

That at

6 L	
1.85	
3.15	
6.70	
8.35	

The Hedge is distant from your Chain-line

6 L
0.50
0.65
0.00
1.00

all which set down in your field Book & being come to your Beacon at C make there a Triangle like unto that which you set out in the field setting the like Marks & Numbers to it.

And then

Field Book

Offsets	6 L	
0.75	0.00	From B to C
0.60	0.50	
0.50	1.05	
0.30	1.75	
0.75	3.40	
0.25	4.80	
0.00	7.15	
0.00	0.00	From C to D
0.50	1.85	
0.65	3.15	
touch	6.70	
1.00	8.35	
0.00	10.00	
0.00	0.00	From D to E
0.25	2.30	
0.55	4.80	
0.00	4.80	
0.00	0.00	From E to F
0.60	0.65	
0.35	2.35	
1.35	3.50	
0.60	4.90	
0.00	7.20	
0.00	0.00	From F to A
0.55	1.75	
touch	2.85	
0.60	5.10	
0.110	7.15	
0.25	7.55	
0.35	8.10	
0.00	9.30	

4. Begin to measure from the Beacon at b towards that at D in a right line & in your measuring along you will find

That at { 2.30 / 4.80 } The hedge is distant from your chain line { 0.25 / 0.55 } all which set down in your field Book. & then

5. Begin to measure from the Beacon at D towards that at b where you find

That at { 0.65 / 2.35 / 3.50 / 4.90 } The Hedge is distant from your chain-line { 0.60 / 0. 35 / 1. 35 / 0.60 } all which you must set down in your field book. & so proceed,

6. Measure from the Beacon at b to the Beacon at A where you began, & in your progress you find

That at { 1.75 / 2.85 / 5.10 / 7.15 / 7. 55 / 8. 10 } The Hedge is distant from your Chain line { 0.55 / 0.00 / 0.60 / 0.40 / 0.25 / 0.35 } all which set down in your field book in the Order you see them set in the Exemplary Table in the Margin.

and thus having finished your Work in the field you may proceed to protract or lay down the same upon Paper & make a fair Plot thereof.

fig: 1 at the Beacon B.

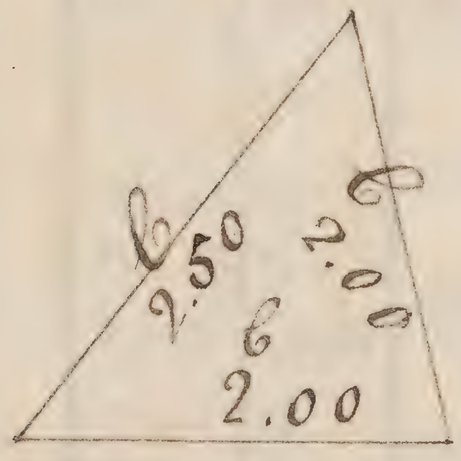
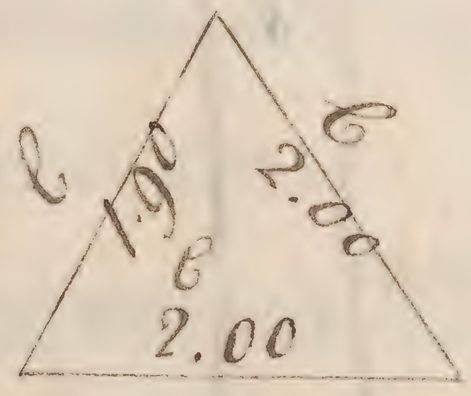


fig: 2 at the Beacon b.



How to protract & make a fair Draught or Plot of the Wood before measured.

Being provided with a Sheet of Paper or Parchment upon it draw a right line at pleasure: & laying your field Book before you, therein you will find that from your first Beacon at A to your second at B there was contained 7 Chains 15 Links: therefore,

1. Take 7 C. 15 L. from your Scale & prick that Distance down (upon your Line before drawn) from A to B: And because the Beacon at a was distant from the hedge 75 L. take 75 L. from your Scale & set them from A to 1. Then at 0 C. 50 L. from a the Hedge is distant from the Chain Line 60 L. therfore take the 50 L. & set them from A to a, & the 60 L. from a to 2. Then

At	1 C. 0 L.	at	A	the Hedge	0 C. 50 L.	Set	B	to	3	and	1 . 2
	1 75		C	is distant	0 30		C		4	draw	2 . 3
	3 40		d	from the	0 75	which	d		5	the	3 . 4
	4 80		e	Chain line	0 25	from	e		6	Lines	4 . 5
											5 . 6

& so have you finish'd that Side of the Wood that lies against the Chain-line a. B. then

2. When you come to your Beacon at B have recourse to your field Book & upon B make a Triangle equal to that you measured out in the Field, by setting 2 6. from B to ☉ & from B to ☿, & from ☉ to ☿ 2 6. 50 L. and drawing the line B ☿ 6, which by your Book you will find to contain 10 6. Then

3. going from B towards 6 you will find by your field Book, that

at	16	85 L.	at	f	The Hedge	0 6. 50 L.	Set	f	to	7	& draw 6. 7
	3	15		g	is distant	0 65	which	g		8	7. 8
	6	70		h	from the	0 00	from	h		9	the lines 8. 9
	8	35		i	Chain line	1 00		i		10	9. 10

so shall you have protracted that part of the Wood that lies against the Chain line B 6 then

4. When you come to your Beacon at 6 having Recourse to your field Book again, upon the point 6 make a Triangle equal to that you measured out in the Field by setting 2 6 00 L from 6 to * upon the line B 6 & from 6 to 5 upon the Line 6 D with the Distance of 16 90 L between them & through the point 5 draw the Line 6 D which (by your Field Book) contains 4 680 L. And then

5. Measuring from your Beacon at 6 towards the Beacon at D you will find by your Field Book that

at	26 32	at	k	the hedge was	0 6 25	set which from	k	to	11	& draw	10 . 11
	4. 80		l	distant from	0 55		l		12	the lines	11 . 12
				the Chain line							

so shall you have protracted that part of the Wood that lies against the Chain line 6 D.

6. When you come to the Beacon at D look for the length of your next Chain line from D to E which you will find to contain 7 6.20 L which take in your compasses & setting one foot in D, with the other foot describe the obscure Arch H H: Also the length of your Chain line from the Beacon at E to the first Beacon at A you find to be 9 6.30 L which also take out of your Scale & setting one foot of the compasses in a with the other describe the obscure Arch K K, crossing the other Arch in the point & then draw the two lines D E & A E; & so are all your Chain lines protracted. And then

7. You will find by your Field Book that in measuring from D to E you found that

at	06.65 L	at	m	The Hedge	06.60 L	set	m	to	13	& draw	12.13
	2 35		n	was distant	0 35		n		14		13.14
	3 50		o	from the	1 35	which	o		15	the lines	14.15
	4 90		p	Chain Line	0 60	from	p		16		15.16

& so you have protracted that part which lies against your Chain line between D & E.

8. Lastly, In measuring from the Beacon at E to the first Beacon at A you do find by your field Book, that

at	16.75 L.	at	q	the hedge	06.55 L		q	to	17	& then	16.17
	2.85		r	is distant	0.00	set	r		18		17.18
	5.10		s		0.60		s		19		18.19
	7.15		t	from the	0.40	which	t		20	draw the	19.20
	7.55		u		0.25		u		21		20.21
	8.10		x	Chain line	0.35	from	x		22	lines	21.22

& so is your Protraction ended. The form of the Wood being as in the Figure.

In the casting up whereof you must first find the content of the whole figure contain'd by the chain lines AB, BC, CD, DE, & EA: And afterwards of the Triangles & Trapezias made by the Off-sets; which being deducted from the content of the whole figure ABCDE, there will remain the Area or Content of the Wood only.

3.05 × 1.80 = 5.4900 = a

1.30 × .30 = .3900 = b

.60 × .50 = 1.10 × .35 = .3850 = F

3.00 × .60 = .80 × .20 = 2.4000 = G

2.50 × .55 = .75 × .20 = 1.250 × .75 = 1.750 = 1.8750 = O

4.90 × .50 = .80 × 30 = 3.9200 = 80 × R

C. L
5.490 = a
3.311 = B
0.390 = C
0.300 = D
1.162 = E
0.385 = F
2.400 = G
1.320 = H
0.675 = I
2.820 = K
4.650 = L
1.920 = M
0.540 = N
1.875 = O
3.300 = P
1.560 = Q
3.920 = R
36.018

4.30 × .55 = .77 × .22 = 3010 × 27 = 3010 = 3.3110 = B

.45 × 1.55 = .30 × .75 = .75 × 775 = 1085 = 1.1625 = E

2.20 × .60 = 1.3200 = H

1.00 × .55 = 1.55 × 3.00 = 4.6500 = L

2.20 × 1.50 = 3.3000 = P

1.35 × .50 = .6750 = I

2.35 × .55 = 1.20 × .65 = 1.20 = 28200 = K

2.40 × .50 = .80 × .30 = .80 = 1.9200 = M

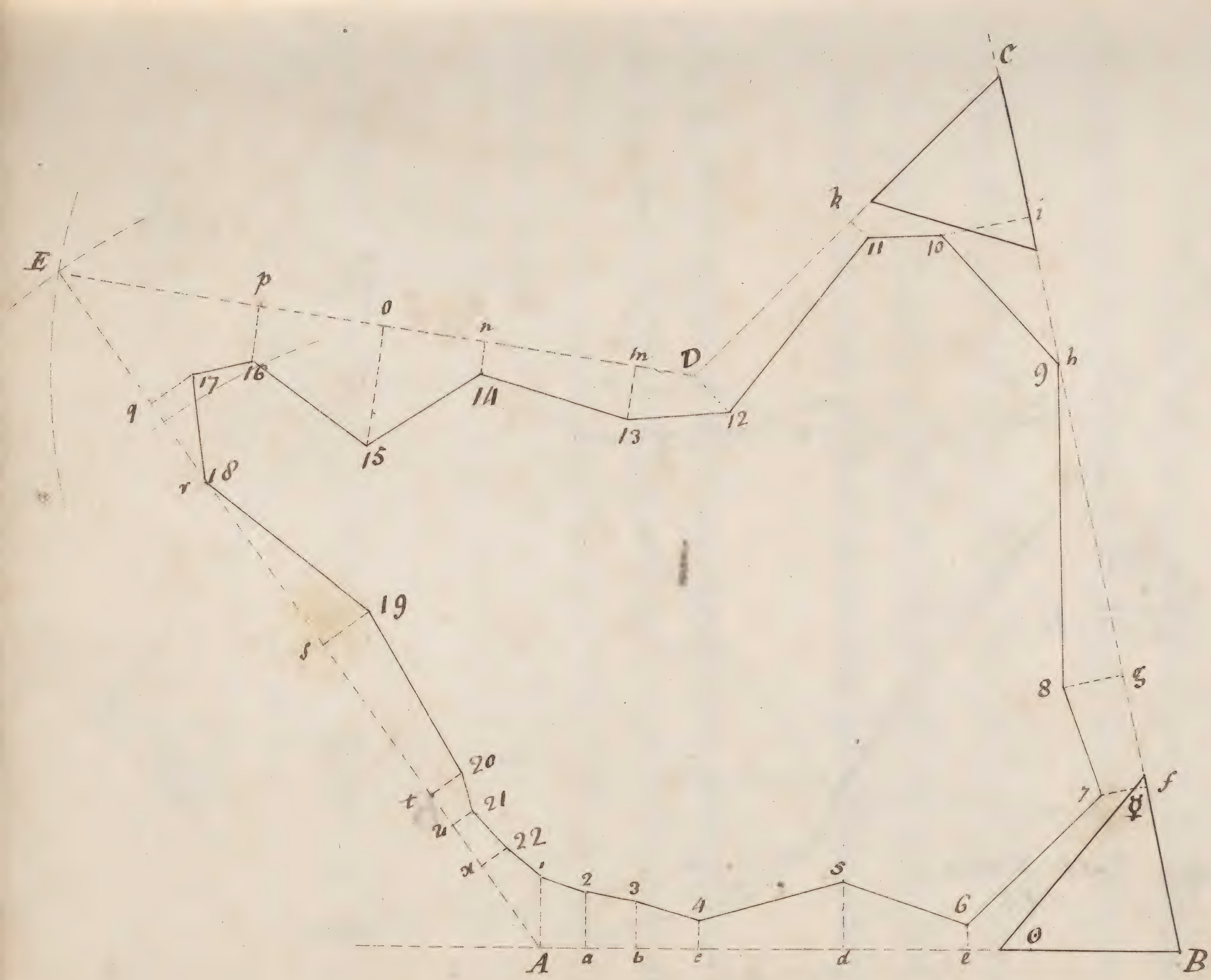
1.95 × .80 = 1.5600 = Q

.45 × 1.20 = .5400 = N

1.30 × 6.25 = 7.55 × 10.90 = 67950 × 7550 = 82.2950 × 45.5700 = 127.8650 − 36.018 = 91.8470

9.30 × 4.90 = 83700 × 3720 = 45.5700

2 | 91.8470
10 | 40..9235
Acre 4.59235 × 4 = Roods 2.36940 × 40 = Perc. 14.77600


C
k
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11
10
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t
21
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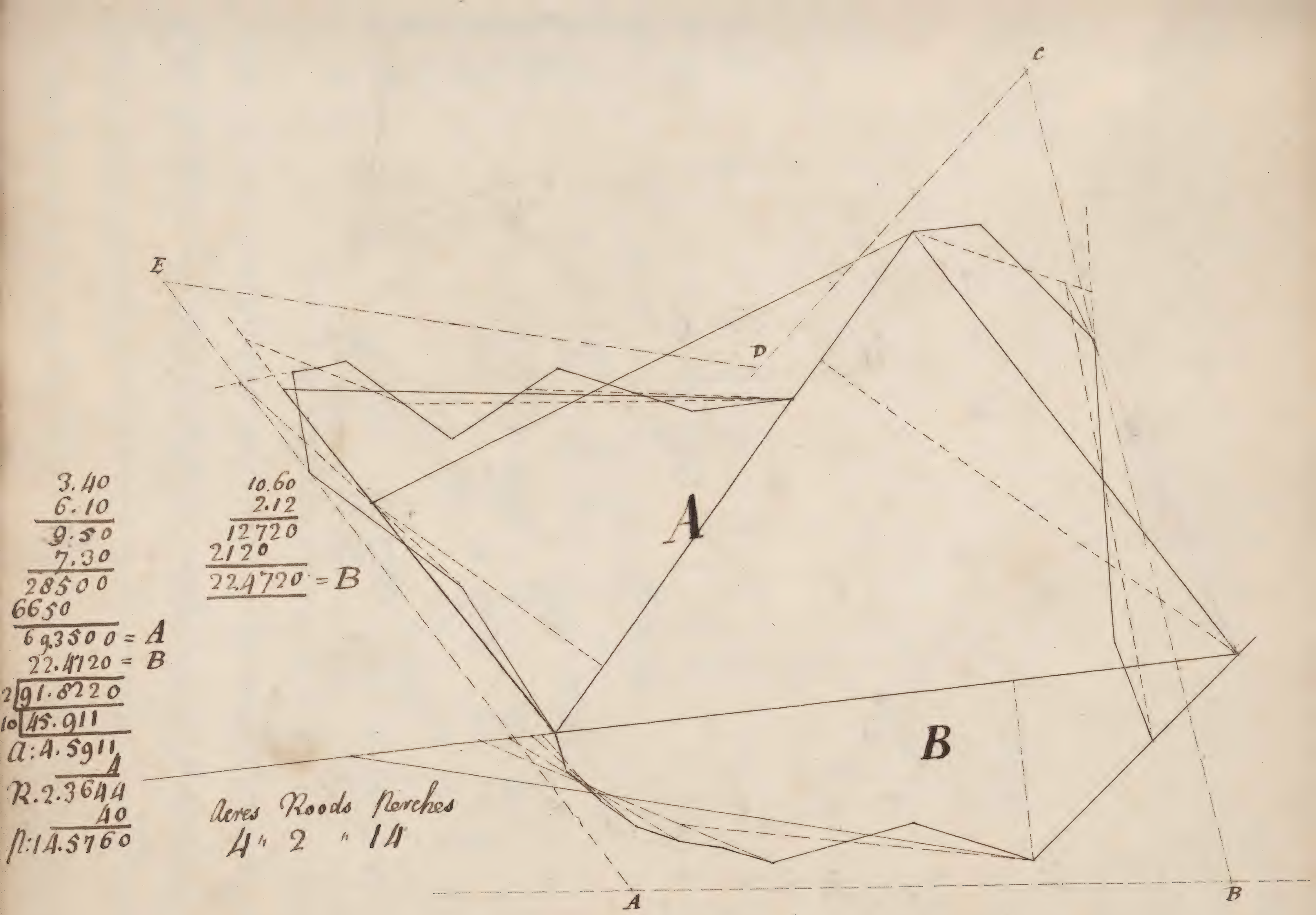
g
7
f
A
a
b
c
d
e
B



3.40
6.10
9.50
7.30
28500
6650
69.3500 = A
22.4720 = B
2 | 91.8220
10 | 45.911
a: 4.5911
A
R. 2.3644
40
ſ: 14.5760

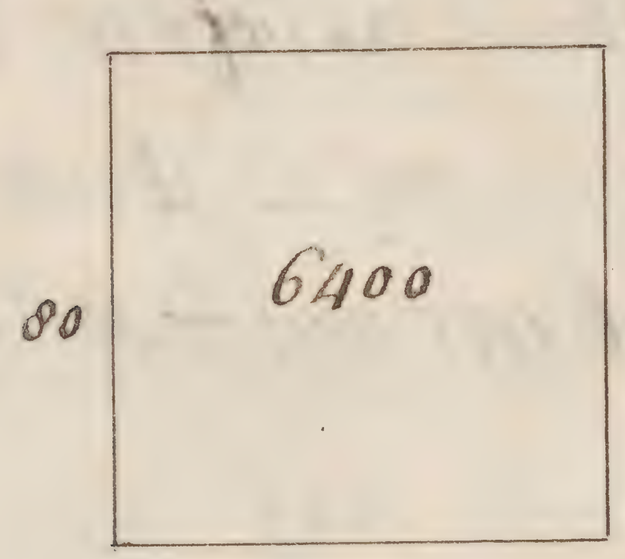
10.60
2.12
12720
2120
22.4720 = B

Acres Roods Perches
4 " 2 " 14

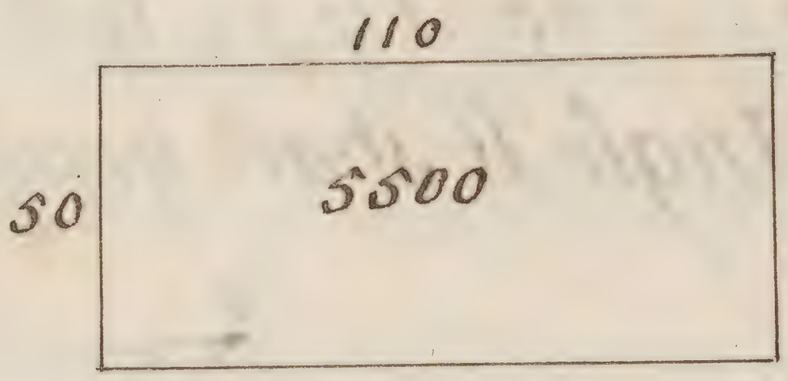
Several Errors in measuring of Land, frequently practised (& too often trusted to) by the Vulgar, detected.

1. Some there are who say, That if two pieces of Land are of an equal Number of Poles Perches (or other Measure) about that those two pieces of Land do contain equal Quantities of Ground.

For the Confutation of this Error Suppose two pieces of Land the one of a true Square form each Side whereof is 80 Perches: And the other a Parallelogram whose length is 110 Perches. & the Breadth 50 Perches, Quare their Area's.

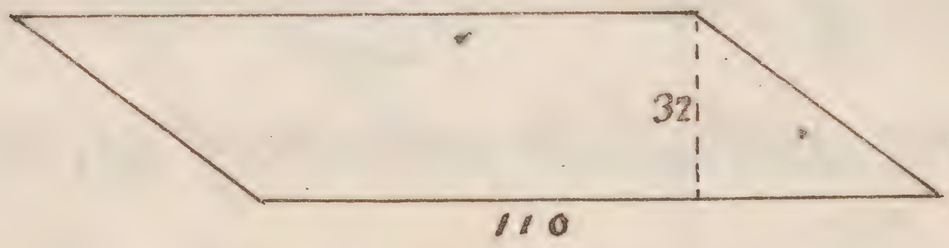


80
80
6400



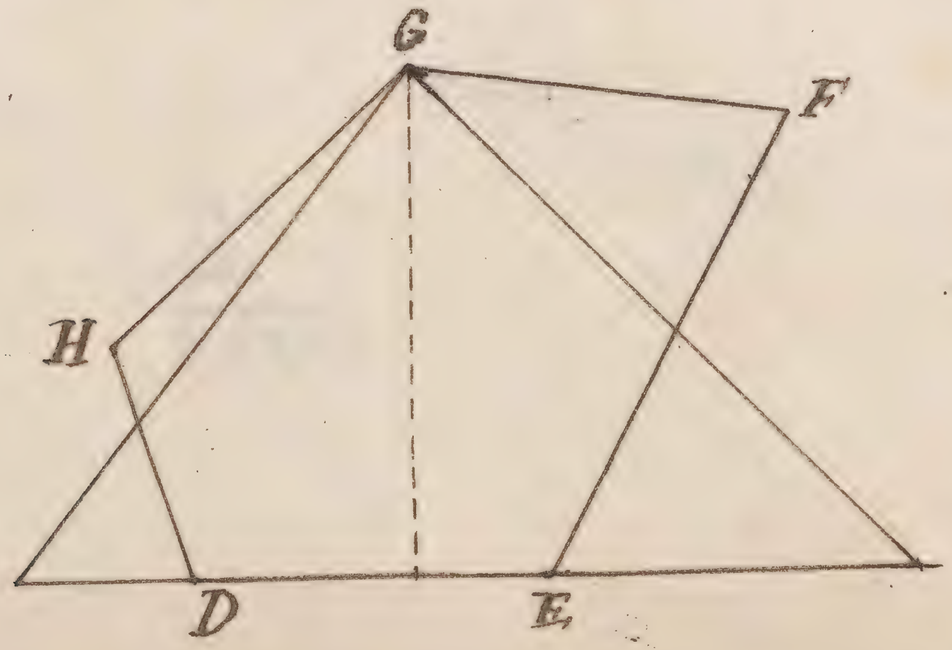
110
50
5500

Cut if a piece of Land of the same length in the Sides & of the same breadth at the Ends whose opposite Sides are parallel having two Angles obtuse & two acute the Error will then be greater for Suppose the Perpendicular Distance of the two longer Sides to be 32 & the Length of the Sides as in the Parallelogram above.



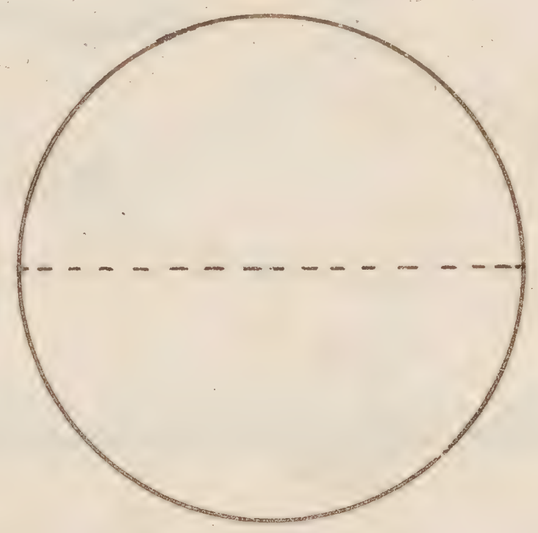
110
32
3520 - Area

Again. Let the irregular piece of Ground D E F G H contain the same Number of Perches about as before Viz: D E 57, & F 85, F G 72, G H 40, H D 66. to find its Area. But if the Angle H shall turn inward the Error will be still greater.



146
82
292
1168
2 | 11972
5986 = Area

Lastly Suppose the Circumference of a Circular piece of Land to be 320 perches about you will find the Error by this to be very great.



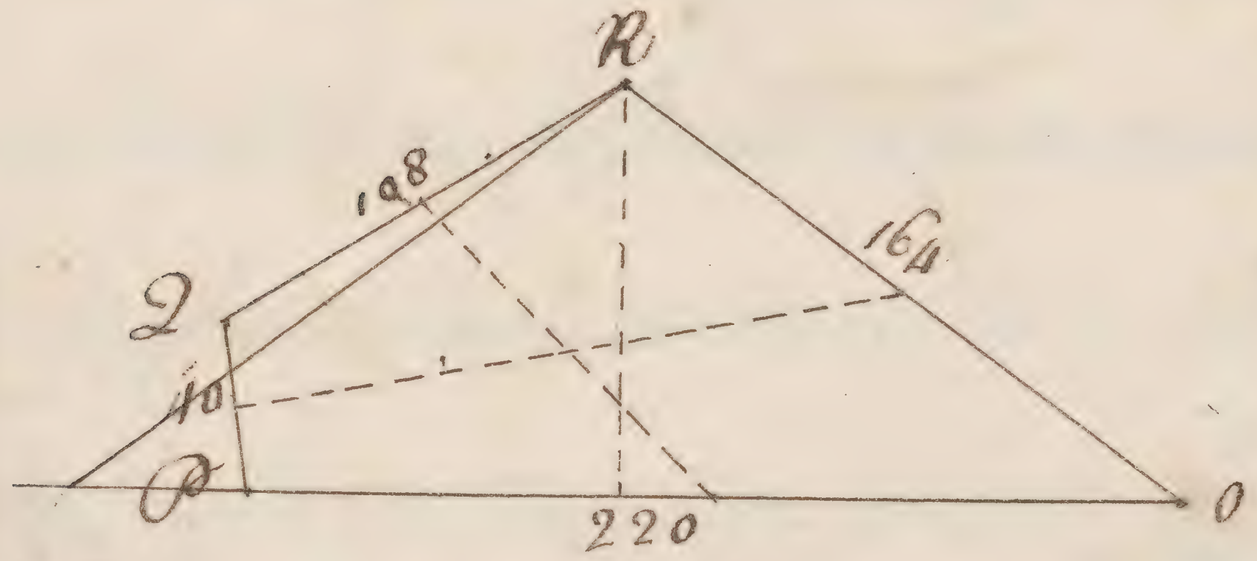
320 = C
320 = C
6400
960
102400 = C^2
.079577
716800
716800
512000
921600
716800
8148.684800 = Area

.3183
320 = C
63660
9549
101.85600 = D

Another Error frequently practised in measuring of Ground (especially for Ploughing, Sowing, Mowing, Reaping &c.) is this: That if a piece of Ground consisting of four Sides (how unequal soever they be) if you add the two opposite Sides together & take their halves those two halves multiplied togeth shall be the true Content of that piece of Ground.

Others there are that say, That if a four-sided piece of Ground be measured cross from the middle of one Side to the middle of the other Side both ways & multiply those two Lengths together shall give the true Content of that piece.

For a confutation of this Error Let there be a piece of Land of four unequal Sides as 0 pl 2 R whose Side pl 2 is 110, 0 R 164, 2 R 108, & pl 0 220 perches to find the Area.



163
97
1141
1467
15811 = Area

108
220
2 328
164
164
656
984
164
26896 = Area

164
Ao
2 204
102
102
204
1020
10404 = Area

224
96
1344
2016
2 21504
10752 = true Area

Of the Saying out of Lands.

The Problems herein as met with in Practice are in Number three.

Viz
1. Either the Partition line is required to be parallel to some other line assign'd; or
2. To pass through a Point assigned in the Fence; or
3. To pass through some Point assigned within the Land.

Before you proceed to their Solution find how many Square links are contained in the Number of Acres Roods & Perches by this Analogy.

Viz. As the No. of Square Perches in one square Chain Viz. 16
Is to the No. of Square links in the same Viz. 10000.
So is the No. of Acres Roods & Perches proposed
To the No. of square links contained therein.

Problem 1.

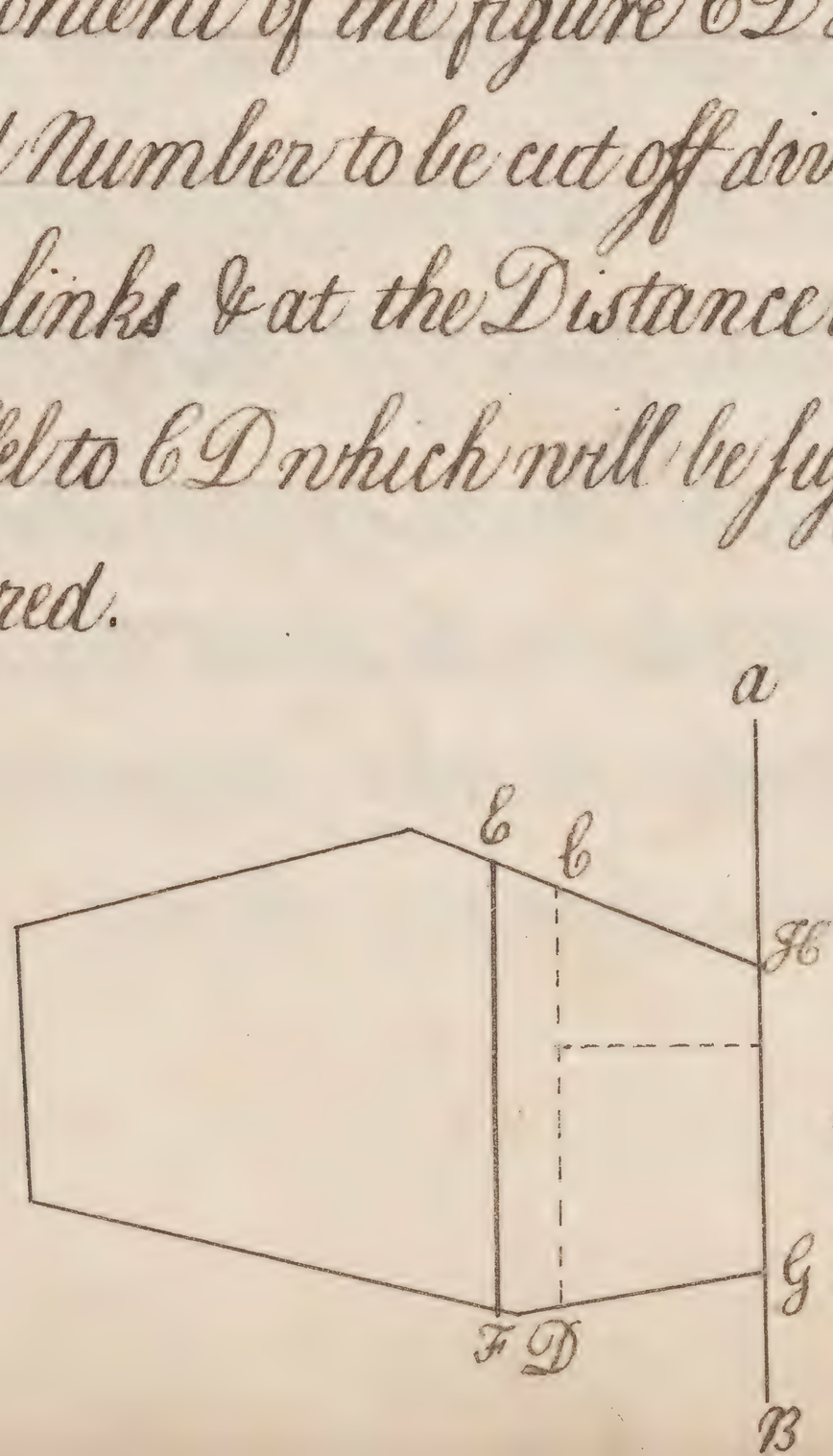
Let it be required to cut off towards a B, $\overset{a}{8}\overset{R}{3}\overset{P}{18}$ by a line parallel to a B?

First draw a line 6 D by guess, parallel to a B & then cast up the Content of the figure 6 D H 6 which if less than the proposed Number to be cut off divide the difference by the length of 6 D in links & at the Distance of the Quotient in Links draw E F parallel to 6 D which will be sufficiently near the Partition line required.

a R P
8 " 3 " 18
4
35
40
1400

16 : 10000 : 1400
10000

4 | 14000000
4 | 3500000
875000



900
1230
2 | 2130
1065
600
639000 = 6 D H 6

639000 | 230
1230 | 2778 *
11300
11070
2300
1230
10700
9840
860

875000
639000
236000 | 191.8 *
230

* the Dist: F D.

But if by Curiosity you are led to correct the Variation; you may conceive 6D to be a line given in position & the Difference as before to be the Quantity to be cut off & & F the line drawn by guess

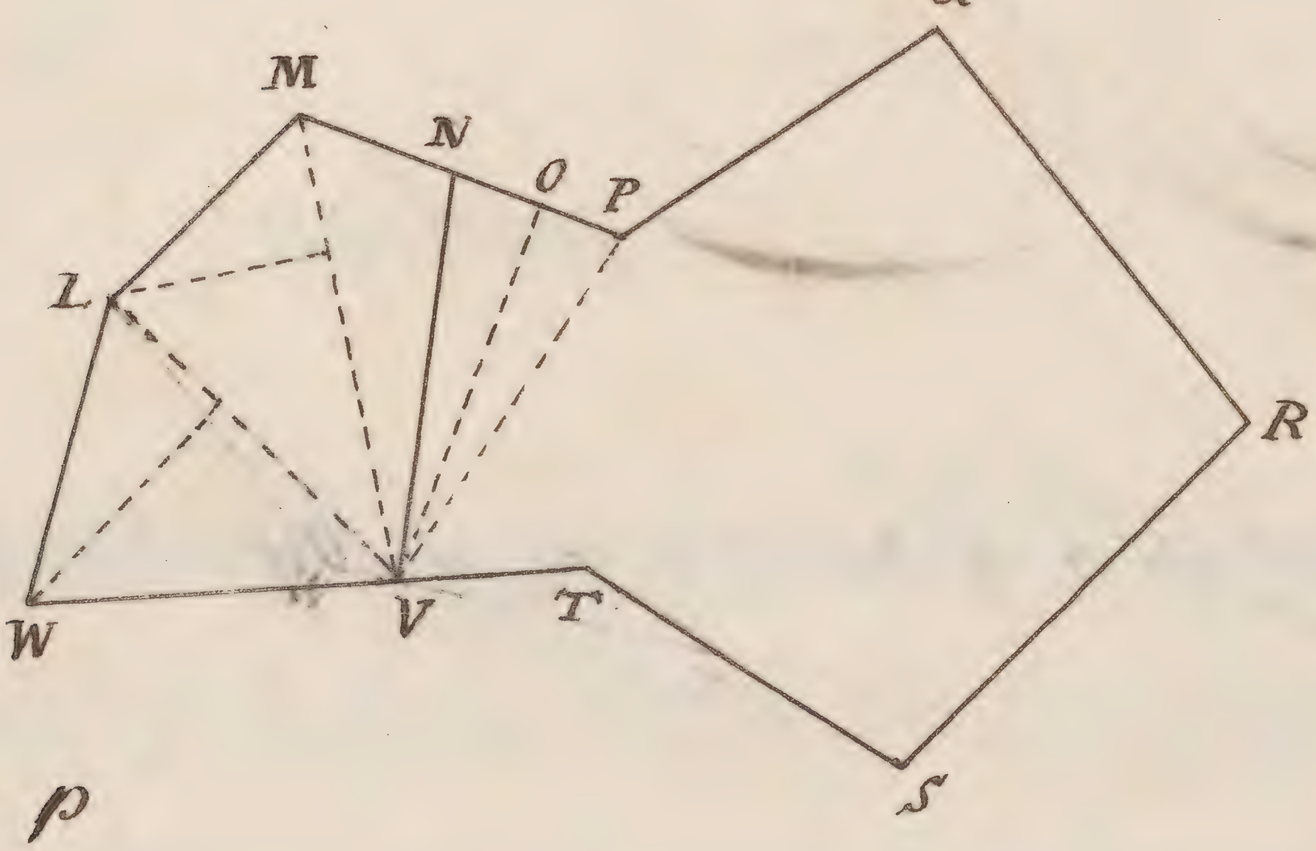
But if the line 6D had cut off the Quantity 6D H G greater than that required then the Partition Line had been more towards G Nwhose Distance from 6D would be found as before by dividing the Difference by the length 6D.

Problem 2.

To cut off $\overset{a\ R\ P}{5 \cdot 0 \cdot 35}$ towards L by a line drawn from the point V.

Reduce the given quantity $\overset{a\ R\ P}{5 \cdot 0 \cdot 35}$ to square Links & they will be 52187 5. Then draw L V forming the Triangle V L W which being cast up amounts to 3800 square links which is too little. To the next angle draw V M, forming the Triangle V L M which being cast up will be found to amount to 297632 square links which added to the Triangle V L W you have 328432 square links the Content of the Trapezium V W L M which is still too little. To the next Angle draw V N forming another Triangle whose Content 110385 0 Square Links added to 328432 Square links the Content of the preceding Fi-

gure gives 73282 square Links which is more than the required Quantity by 210407 square Links; therefore the partition Line passes between pom. Divide the Excess by 497 half the length of VO the perpendicular in Links; & lay 423 the Quotient from P to N; & so shall Vn be the true Line of partition.



a R P
5 . 0 . 35
4
20
40
16 : 10000 : : 835
835
2 | 8350000
8 | 4175000
521875

30800 = VWL
297632 = VLm
403850 = VmO
732282
521875
497) 210407 (423 the Dist. pN
1988
1160
994
1667
1491
166

Problem 3.

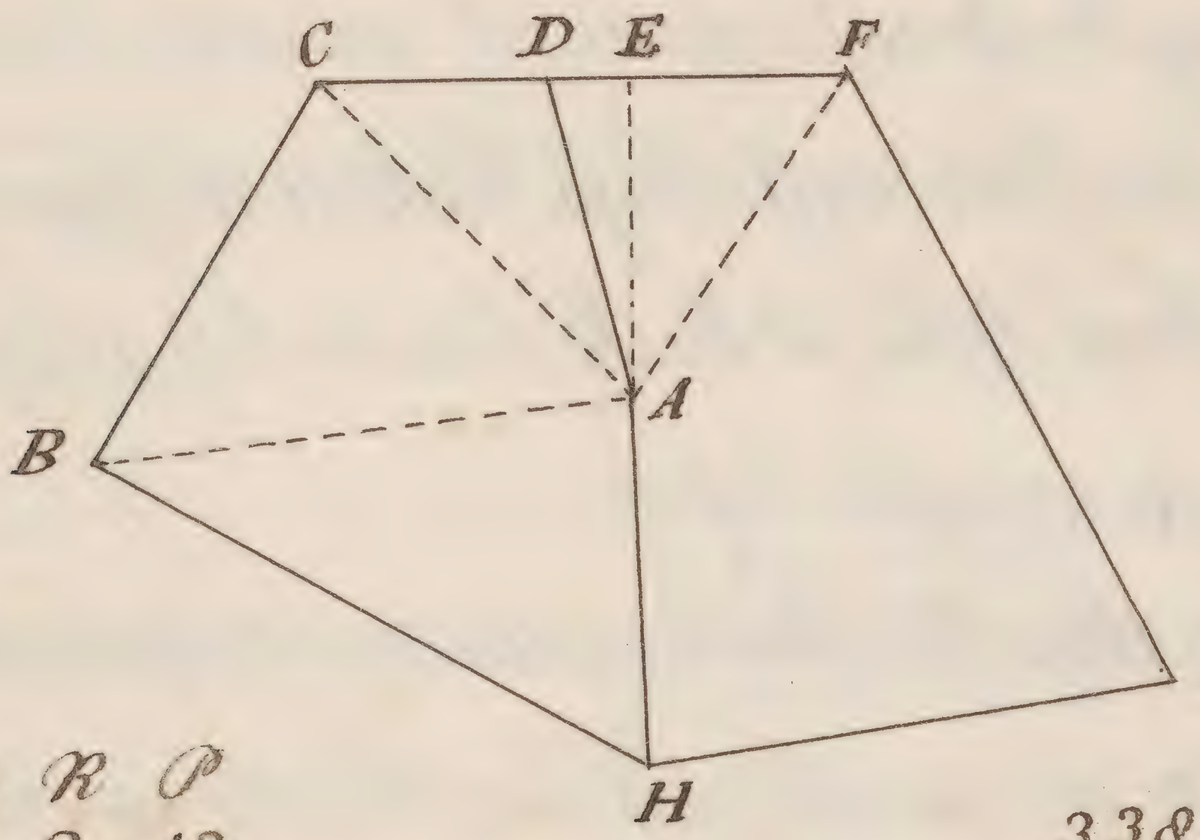
To cut off $\overset{a}{7}\overset{R}{.}\overset{P}{3.13}$ towards B from a Line passing by the point A.

This may be thus performed; consider through which two sides the partition-line will pass. Reduce the figure to a Trapezium still retaining those Sides. Which divide by a Line through the assigned point A as taught before.

But because this Method is tedious & depends on the concourse of many Lines; & tho' we should use Numbers we should little mend the matter & indeed is often impossible to be performed by one single Line, it may not be amiss to shew how to do it by two Lines.

From A to any two Angles H & B draw the Lines A H, A B, forming the Triangle A H B which being cast up amounts to 338000 square links less than the quantity required.— To the next Angle

& dran A & forming the Triangle ABC whose content (being cast up) is 322500 square links which added to the Triangle AHB gives 660500 square Links which are still less than the Quantity required.— To the next Angle F dran A F forming the Triangle A & F whose content 280800 square Links added to the preceding Triangles AHB, ABC gives 941300 which exceeds the quantity required by 158175 square Links now divide this Excess 158175 by 292 half the length of the perpendicular & L & lay the quotient 541 from F to D & dran AD; & so will the Lines HA, AD be the partition required.



D

		a	R	P
16	: 10000 : :	7	3	13

4

31

40

16 { 2 | 12530000
8 | 6265000

783125

338000 = AHB
322500 = ABC
280800 = A & F

941300

783125

292) 158175 (541 = FD

1460

1217

1168

495

292

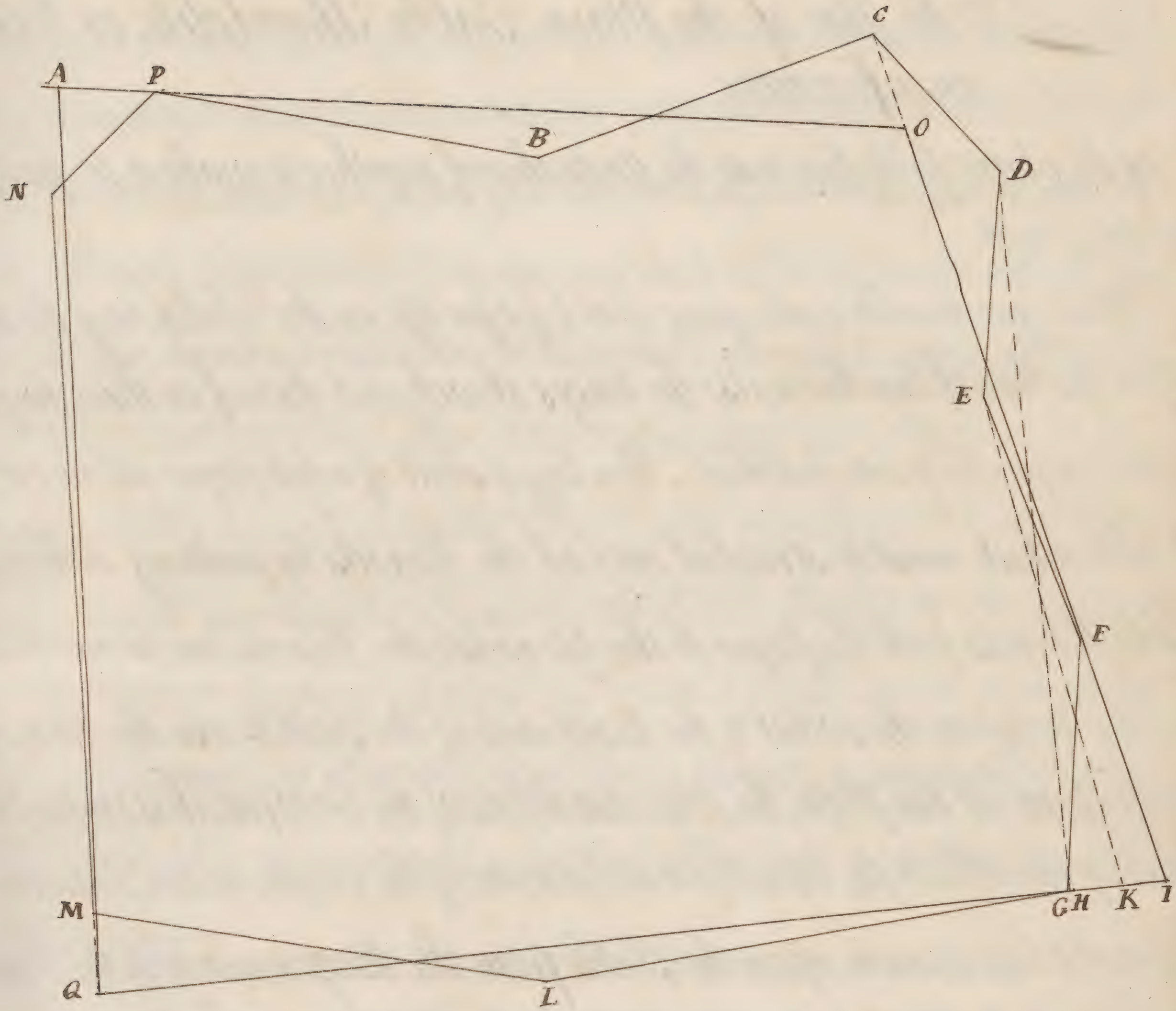
203

It may not be improper perhaps to add in this place the manner of casting up such fields as consist of many small Breaks in the Hedges; without reducing them to a multitude of Triangles. e.g.

Let B C D E F G L M n p be such a Field. Produce N M some one of the longest sides: then lay the edge of a straight Ruler from M one of the Angles at the end of N M to G the next but one holding the Ruler thus fast, take with a Pair of Compasses the Distance from L to the Edge of the Ruler & with this distance let one point of the Compasses move gently close to the Ruler while the other point traces out a line parallel to it & crosses M N in 2, Now draw G 2 & it will reduce that side of the figure which was bounded by the two Lines M L, L G, to another bounded by G 2 one single Line only. In like manner 2 G being produced & a Ruler laid from G to E carry the distance of F from the Ruler parallel to it till you cross M G in K. Secondly lay a Ruler from K to the next point D & carry the Distance of E the last point from the Ruler parallel to it till you cross M G in K. Thirdly lay a Ruler from K to the next point C & carry the Distance of D the last point from it parallel till you

cross MG in I. Lastly draw IC & the Side GC which consisted of the four lines GF, FE, ED, DC, will be reduced to the Side IC consis=ting of one line only. And in like manner we might proceed if the Lines were never so many. And thus laying a Ruler from CP draw PO; & in like manner AQ. So will the ten sided figure be re-duced to a four sided one & so then may be cast up by one Multiplication only.—— Provide a plate of thin Brass in form of an Arch of a Circle, near whose ends let there be drilled small holes; through which string it with a very fine hair. Being thus provided; when a hedge ab-bends in & out in several places & those Bends contain very small Spaces; lay the hair over it lengthwise so that the quantities cut off from the Figure thereby may be equal to those added to it; & with your protracting pin near the ends of the hair make two Marks through which draw a straight Line; and so will this irregu-lar Side be reduced to a Regular one. And here we may observe that in very small bends by the Eye we judge better than we can by the Compasses.

Lastly if hedges consist of large Curvatures chuse out such points & so many of them that right Lines drawn from point to point may vary the Quantity by such quantities only as may be rejected. And herein your hair will be a ready assis-tant.


A
P
B
C
O
D
N
E
F
M
Q
L
G
H
K
I

The Use of the plain Table, Theodolite, or Circumferentor.

Of the plain Table, how to set the parts thereof together & make it fit for Use in the Field.

When you would make your Table fit for Use in the Field, lay the three Boards thereof together, & also the ledges at each end thereof in their due places according as they are marked. Then lay a Sheet of white paper all over the Table which must be stretched over all the Boards by putting on the Frame which binds both the paper to the Boards & the Boards one to another. Then screw on the Socket to the Back-side of the Table, & also the Box and Needle in it's due place, the Meridian line of the card (which is in the Box) lying parallel to the Meridian or Diameter of the Table; which Diameter is a right Line drawn upon the Table from the Beginning of the Degrees through the center & so to the end of the Degrees. Then put the Socket upon the head of the Staff & there screw it: Also put the Sights upon the Index & lay the Index upon the Table. So is your Instrument prepared for Use as a plain Table or Theodolite the Difference on-

by being in placing of the Index. For when you use your Instrument as a Plain Table you must pitch your Centre in any part of the Table which you shall think most convenient for the bringing on of the work which you intend; But if you use the Instrument as a Theodolite then the Index must be turned about upon the Centre of the Table; for which purpose there is a piece of Wire which goes through a small hole of Brass fastned to the Index, & so into the Centre by which means the Index keeps it's constant Place only moving upon the Centre. Your Instrument being thus order'd you may use it either as a Plain Table or a Theodolite: But if you would use it as a Circumferentor you need only screw the Box & Needle to the Index & both of them to the head of the Staff with a brass screw-pin fitted for that purpose; so that the Staff being fixed in any place the Index & Sights may turn about at pleasure without moving of the Staff.

And Now is your Instrument a good Circumferentor. Also when you have Occasion to measure any Altitude hang the Label upon the farther Sight. And thus you are exactly fitted for all Occasions.

How to measure the quantity of an Angle in the Field by the plain Table, Theodolite, & Circumferenter; & also to observe an Angle of Altitude.

1.st You must understand that when the plain Table is mentiond for any Work to be performed thereby is meant the Table when it is covered with a Sheet of Paper, upon which all observations of Angles that are taken upon the Table in the Field do agree exactly in Proportion with those of the field it self but are not denominated by their Quantities but by their Symetry or Proportion.

2.nd When the Theodolite is mentiond to perform any work therewith is meant the Degrees described on the Frame of the Table which supplies the Use thereof.

3.rd When mention is made of the Circumference is meant the Index with the Box & Needle screwd to the Staff.

First how to observe an Angle in the field.

By the Plain Table.

Suppose E K & K G to be hedges or two Sides of a Field including the Angle E K G & that it were requir'd to draw upon your Table an Angle equal thereto

First place your Instrument as near the angular point K as convenency will permit, turning it about till the North-end of the Needle hang directly over the Flower-de-luce in the Box & then screw the Table fast. Then upon your Table with your protracting-pin or compass point assign any point at pleasure upon the Table & to that point apply the Edge of the Index turning the Index about upon that point till through the sights thereof you espie a mark set up near E, but as far from E as the Instrument near K is from E K; & then with your protracting-pin or compass point or black Lead draw a line by the Side of the Index to the assigned point upon the Table.

Then (the Table remaining immoveable) turn the Index about upon the same point & direct the sights to a Mark set up at Lg or parallel thereto that is so far distant from Lg K as your Instrument is placed from K G & then by the Side of the Index draw another line to the assigned point. So shall you have drawn upon your Table two Lines which shall represent the two Hedges G K & K Lg; & those lines shall include an Angle equal to the Angle G K Lg. And although you know not the Quantity of this Angle yet you may easily find the Quantity thereof if there were any need. For in working by this Instrument it is sufficient only to give the Symetry or proportion of Angles & not their Quantities as in working by the Theodolite or Circumferentor it is. Also in working by the Plain Table there needeth no protraction at all for you shall have upon your Table the true Figure of any Angle or Angles which you observe in the Field in their true positions without any farther Trouble.

Here Note, That when I say plant your Instrument at any Angle I mean as near that Angle as you can conveniently come; & when I say direct to any mark in any other Angle I mean a Mark set up near the Angle equidistant from the Hedge with the Instrument.

2.nd By the Theodolite.

Let it be required to find the Quantity of the Angle E K g by the Theodolite place your Instrument at K layeng the Index on the Di-ameter thereof; then turn the whole Instrument about, (the Index still resting on the Diameter) till through the Sights you espie the Mark at E; then screwing the Instrument fast there turn the Index about upon the Centre till through the sights you espy the Mark at g, then note what degrees (on the Frame of the Table) are cut by the Index which you will find to be 114.° & that is the quantity of the Angle E K g.

One thing is to be observed that when you look on the Divisions of the Theodolite which are on the nearest Edge to you, in some the Num-bers encrease towards the left hand in others towards the right: and this is the first thing to be observed if you meet with an Instrument you never saw before. And then standing at the Instrument to take observation if the Degrees encrease towards the left Hand the fixt sights shall be directed along E K the left hand hedge if otherwise the contrary.

3.rd By the Circumferentor.

It is required to find the Quantity of the former Angle E K g by the Circumferentor; first place your Instrument (as before) at K with the Flower-de-luce in the Card towards you; then direct your Sights to

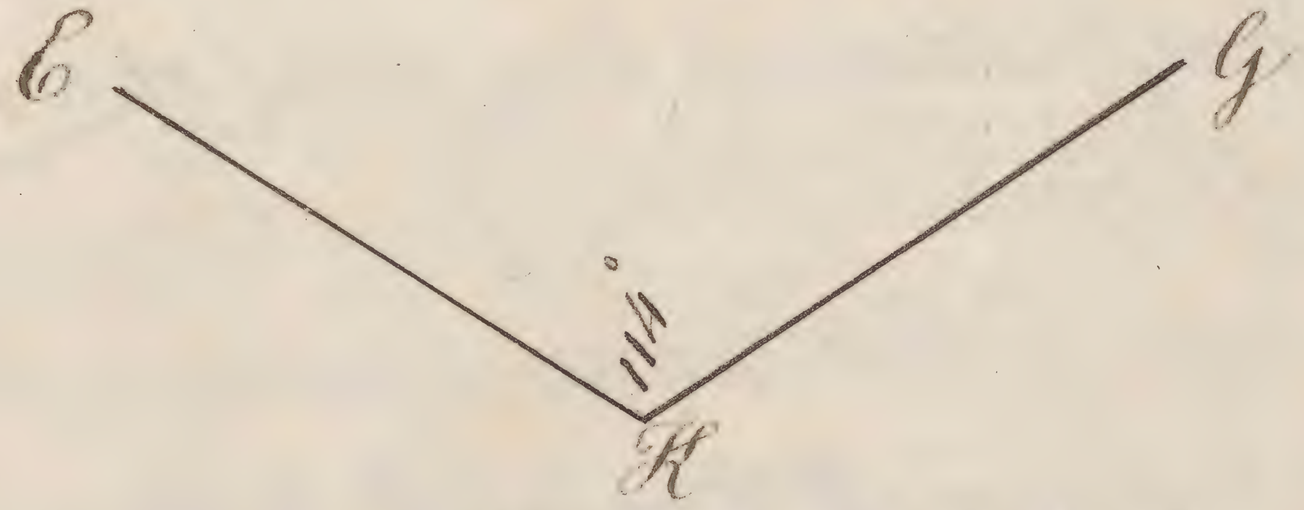
& observe what degrees in the Card are cut by the South end of the Nee-
dle which let be 296: then turning the Instrument about upon the Staff
(the the flower-de-luce always towards you) direct the Sights to ly no-
ting then also what Degrees are cut by the south end of the Needle
which suppose 182. This done (always) subtract the lesser Number
of degrees out of the greater as in this Example 182 from 296 & the
Remainder is 114 degrees which is the true quantity of the Angle K Lg.

Again, the Instrument standing at K & the sights being direc-
ted to L as before suppose that the South end of the Needle had cut
79° & then directing the Sights to g the same End of the Needle had
cut 325.° Now if from 325 you subtract 79 the Remainder is 246°.
But because this Remainder 246 is greater that 180° you must
therefore subtract 246 the Remainder from 360,° & there will re-
main 114° the true quantity of the inquired Angle. And thus
you must always do when the Remainder exceedeth 180°.

This adding & subtracting for the finding of Angles may
seem tedious to some; but here take Notice, that for quick dispatch the
Circumferentor is as good an Instrument as the best. For in going
round the field or in surveying of whole Manour you are not to
take Notice of the quantity of an Angle but only to observe what
Degrees the Needle cutteth which in those cases is sufficient as will
appear hereafter. But in taking of Distances by the Circumfe-
rentor it is altogether necessary as will appear hereafter & for that

reason you are here shew'd how to find an Angle by the Circumferentor, & also that you might thereby perceive what Congruity & Harmony there is in all the three Instruments.

And here we are to observe That the Circumferentor is the only Instrument proper to the mapping of Harbours & drawing the true position of Rocks Lands Shoals Sounding, &c. nor is it scarcely possible to be done by any other Instrument but such as have a Needle & divided Card. But of this more in the Appendix.



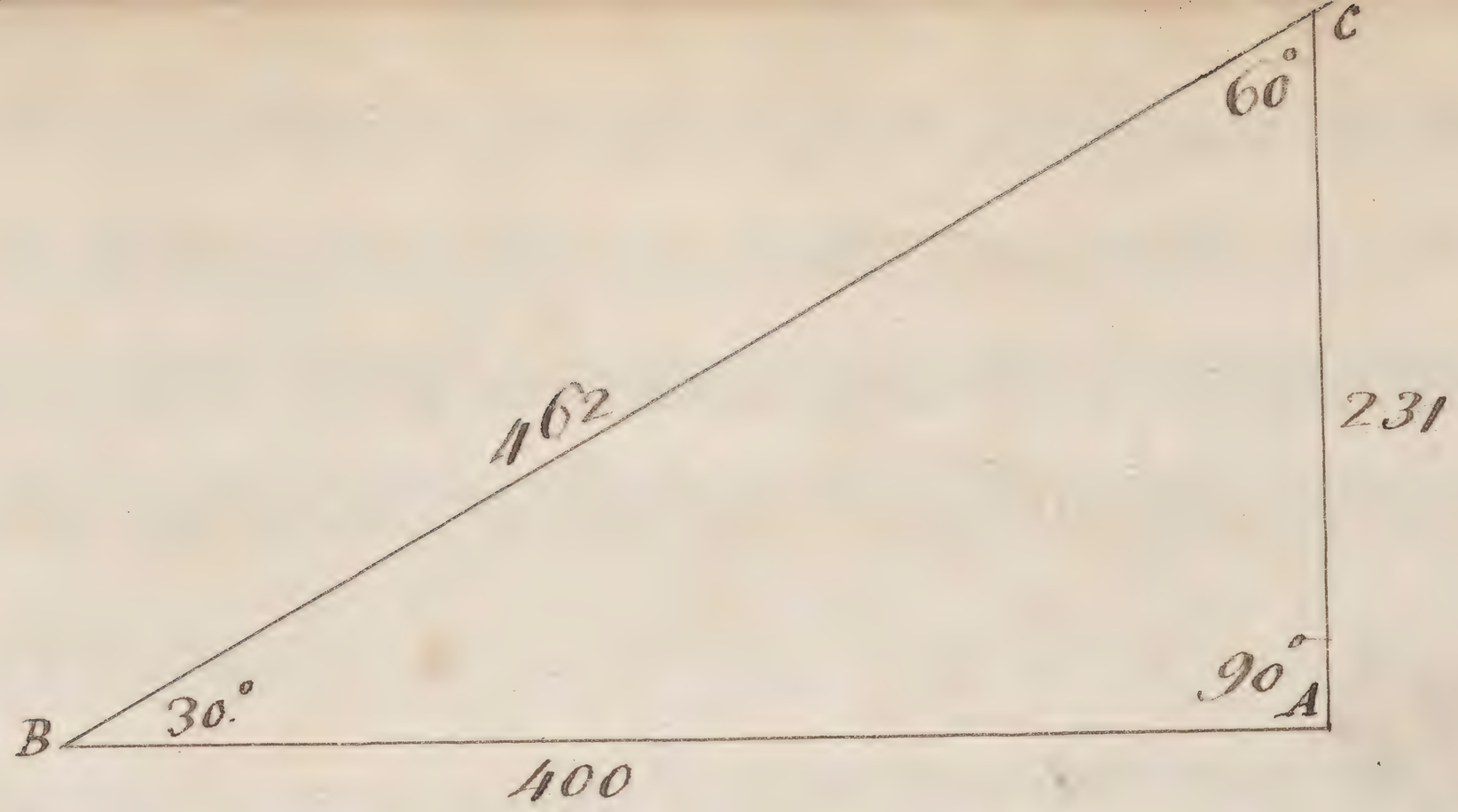
4 How to set the Index & Label horizontal upon the Staff.

When you have screwed the Index & Sights to the Staff as a Circumferentor before you put the Label upon the Brass pin or Wire you must hang a line & Plummet upon that pin & then put on the Label; then move the Index up & down till the thread & Plummet hang directly upon a line which is gaged from under the pin all along the Sight & then doth the Instrument stand horizontal or level, which it must always do when you take an Altitude therewith.

5. How to observe an Angle of Altitude.

The Label which is to be hanged on one of the Sights of the Circumferentor (as was intimated in the Description thereof) & the Tangent-line on the Edge of the Index is only for finding the Angles of Altitude & is therfore only usefull in taking of Heights & in surveying of mountainous & unevenground. The manner how to observe an Angle of Altitude by this Label & the Tangent-line on the Index is this.— Suppose CA be a Tree, Tower, or Hill whose Height is required. Your Instrument being placed at B exactly level direct the Sights thereof towards C A & there fix it hanging the Label on the farthest Sight on a pin for that Purpose. Then move the Label to & fro along the side of the Index till through the Sight at the End of the Label & by the pin on which the Label hangeth you espie the very top of the Object to be measured at C; then note what degree of the Tangent-line is cut by the Label which suppose 30 & that is the quantity of the Angle of Altitude it being equal to the Angle CBA.—

Thus by the Rules in this Chapter deliver'd may the true quantity of any Angle be easily taken; & this is the most convenient Use to be first placed.— I will now shew you how by your several Instruments contain'd in the Planerganon all manner of Heights & Distances may be taken whether accessible or inaccessible several Ways with other necessary Conclusions incident to the whole Art of Surveying.



How to take an Inaccessible Distance at two Stations by the Planorganon: &

1.st By the Plain Table.

You have been taught before how to make Observation of any Angle in the field by the several Instruments contained in the Planorganon & also an Angle of Altitude by the Index & the Label thereunto annexed. It is conceived convenient now to shew how all manner of Heights & Distances may be readily & exactly measured several Ways thereby serving to prove each other, whether they be accessible or inaccessible. And first of Distances.

You may remember a previous Intimation that the measuring a Height or Distance is only to resolve a Triangle; so that when you make any Observation either of Height or Distance, the Observation of Angles which you make are the Angles of some

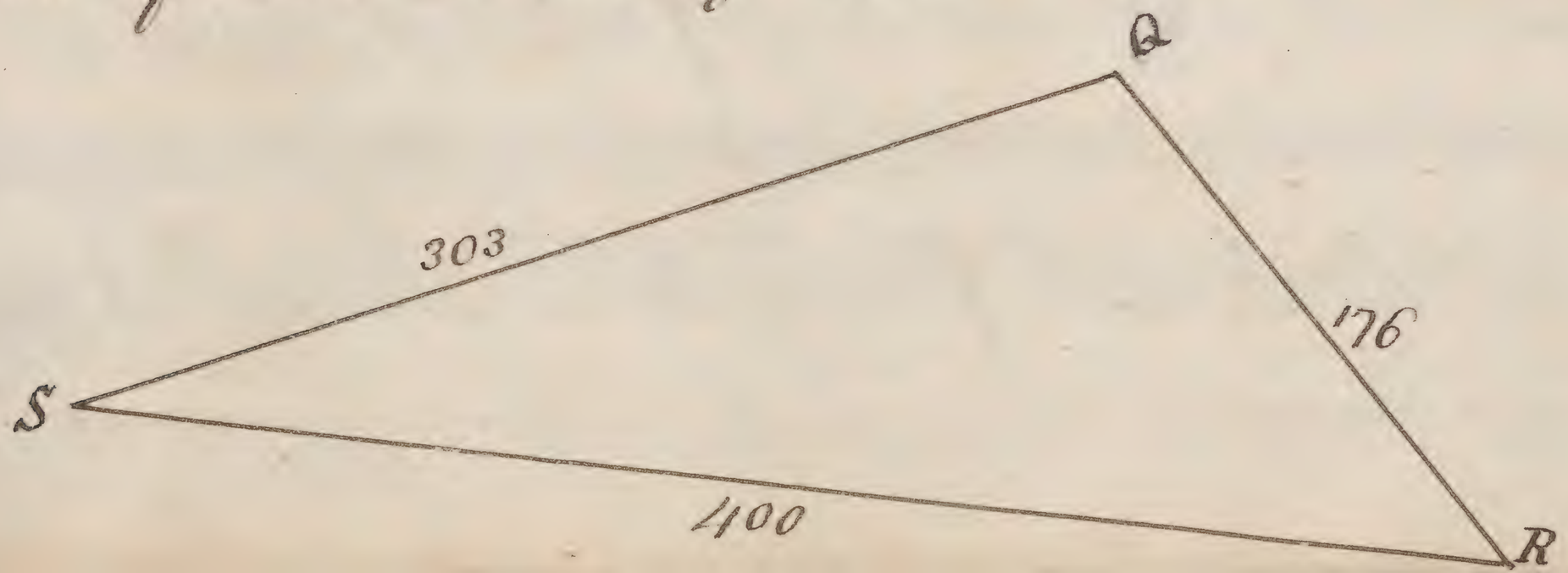
Triangle & the lines which you measure on the Ground are the sides of the same Triangle & these are the given parts of the Triangle.

The manner how to take a Distance by the Plain Table is thus. Suppose you were standing in a field at R & that at S were some eminent Mark (as a Tree, Church, House, or such like) & that it were required to find the Distance between R & S.

First, place your Table at R & thereon assign any point at pleasure, upon which point apply the Edge of your Index turning it about upon that point till through the Sights you espy the Mark at S; & draw the line by the Side of the Index as R S. —— Then in some other convenient place of the field (as at 2) let a Staff or other Mark be erected & the Table remaining as before turn the Index about upon the point R, till through the Sights you espie the Mark at 2 drawing a Line by the Side thereof as R 2, so have you described upon your Table an Angle equal to the Angle 2 R S. Then (with your Chain) measure the distance 2 R which let be 176 feet then take with Compasses 176 out of any Scale & set it upon your Table from R to 2; so shall this point 2 upon your Table represent the mark at 2 in the Field. —— This done set up a Staff at R where your Instrument Stood, & remove your Table to 2 laying the Index upon the line 2 R & holding it fast there turn the whole

Table about till through the Sights you espie the Mark set up at your former place of Standing at R; then screw the Table fast & lay the Index on the point 2 turning it about upon the point 2 till through the Sights you espie the Mark at S then draw a line by the Side of the Index which will cut the line RS (first drawn) in the point S. — By this means shall you have upon your Table a Triangle equal to the Triangle 2RS the correspondent sides & Angles thereof being proportionally equal with those in the field Therefore if with your compasses you take the length of the side RS & apply that distance to the same Scale from whence you took the side 2R you shall find it to contain 400 feet, & that is the distance betwen RS likewise if you take with your compasses the length of the line 2S & apply it to the same Scale you shall find it to contain almost 303 & so many feet is the distance 2S.

In this manner may the distance between any two places be measured, although they be situated that by Reason of Water or other Impediments you cannot approach near unto them. And here Note, That when you take your second Station you take it so large if the Ground will permit that the Angle S may approach to 90° for the nearer S is to 90° the more directly will the point of Intersection be found.



How to take an inaccessible Distance at two Stations by the Theodolite.

In the former Diagram let R & 2 be two Stations from either of which is required to find the distance to S.

First place your Instrument at R laying the Index and sights upon the Diameter thereof turning the whole Instrument about till through the Sights you espie your second Station at 2 & then screw it fast then turn the Index about upon the Centre till through the Sights you espy the Mark at S noting the degrees cut by the Index which suppose 45° 10; then remove your Instrument to 2 laying the Index on the diameter thereof & holding it there turn the whole Instrument about till through the sights you espie your mark at S & fixing the Instrument there turn the Index about till through the Sights you see the

Mark set up at your former Station at R noting the degrees thereat which let be 110° 30'. This done measure the distance of the two Stations 2R which let be 176 feet. So in the oblique-angled Triangle 2RS you have given (1) the angle SR2 45° 10' the angle observ'd at your first Station; (2) the angle R2S 110° 30' which was the angle observed at your second Station; & (3) you have the measured Side 2R 176 feet which is the Distance of your two Stations & you are to find the two other Sides RS & 2S which you may find by the 1.st Case before taught in this manner. For having two Angles SR2 & R2S given you have also the third Angle RS2 given 24° 20' it being the Complement of the other two to 180.° Then to find the other two sides the Analogy is

1.st for the Side 2S.

As the Sine of the Angle 2SR 24° 20'
Is to the Logarithm of the Side R2 176 feet
So is the Sine of the Angle 2RS 45° 10'
To the Logarithm of the Side 2S 303 feet, fere.

2.nd for the Side RS.

As the Sine of the Angle 2RS 45° 10'
Is to the Logarithm of the Side 2S 303 feet
So is the Sine of the Angle R2S 110° 30' (or 69° 30')
To the Logarithm of the Side RS 400 feet; the distance required.

Having been very particular in this Example to render the same very perspicuous shall in the subsequent part be the briefer.

S. 24° 20′ : L. 176 :: S. 45° 10′

9.614944 : 2.245513 :: 9.850745

2.245513

12.096258

9.614944

303 = 2.481314

S 45° 10′ : L. 303 :: S. 69° 30′

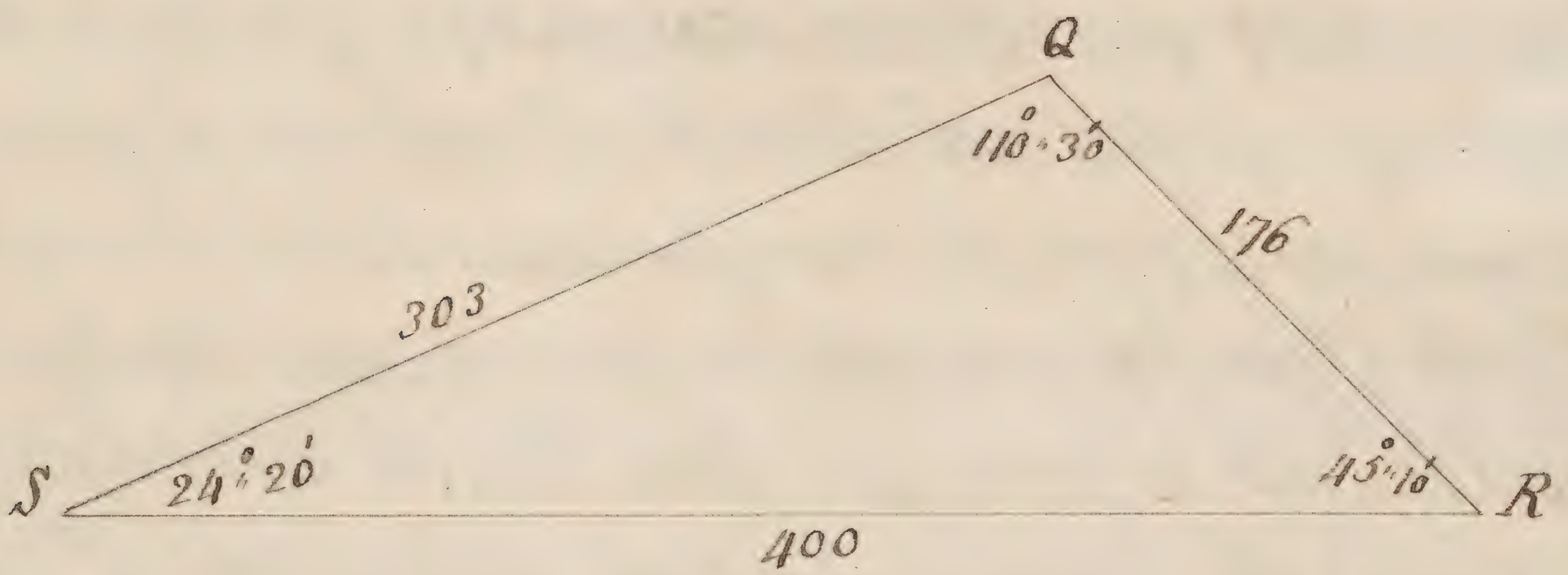
9.850745 : 2.481314 :: 9.971588

2.481314

12.452902

9.850745

400 = 2.602157



How to take an inaccessible Distance at two Stations by the Circumferentor.

Let it be required to find the distance from R or 2 to S.

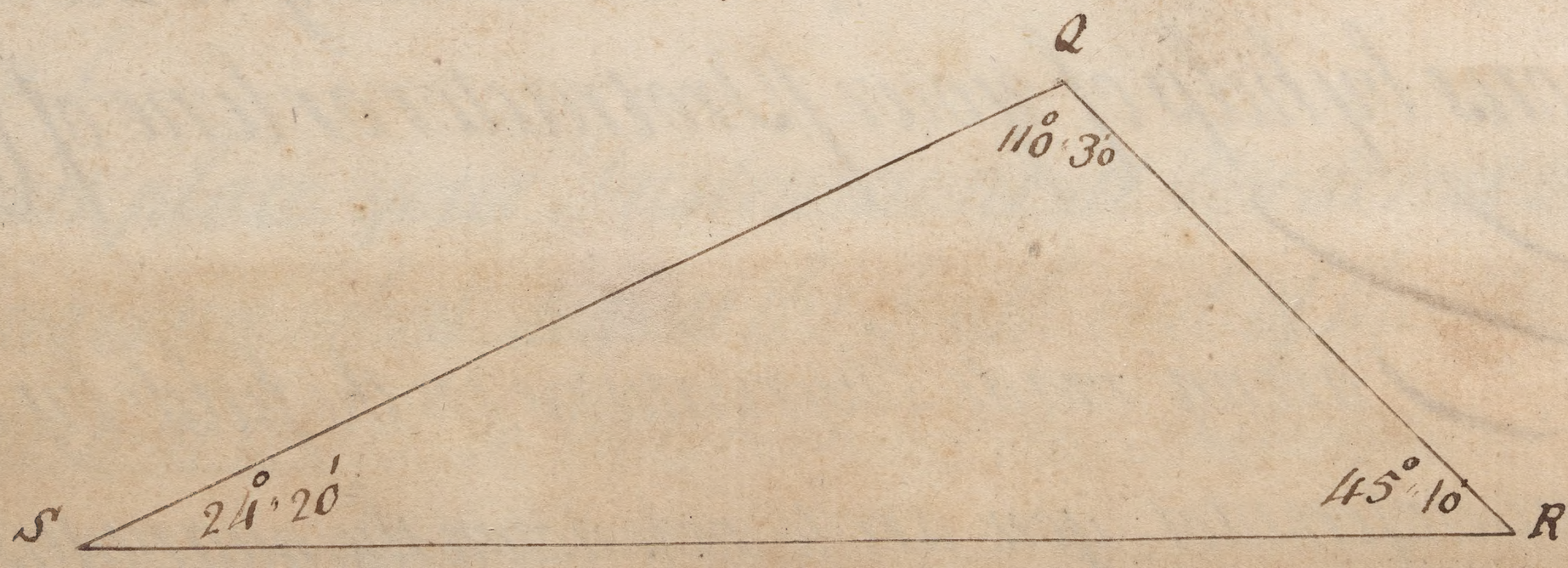
First place your Instrument at R & direct the Sights to S observing what degrees the South end of the Needle cutteth which let be 315°. 30′ then turning the Instrument about

direct the Sights to 2 observing what degrees the Needle there cutteth which let be 270°. 20′; therefore from 315°. 30′ subtract 270°. 20′ & there will remain 45°. 10′ which is the Quantity of the Angle SR2.
Then remove the Instrument to 2 & direct the Sights to R the Needle cutting 90°. 20′; also direct the Sights to S the Needle cutting 339°. 50′; now if you subtract 90°. 20′ from 339°. 50′ the Remainder is 249°. 30′ which (because it exceedeth 180°) subtract from 360°. & there remains 110°. 30′ the true Quantity of the Angle R2S.
Having thus obtained the true Angles R2S & SR2 you must measure the Sationary Distance 2R 176 Feet so have you given in the Triangle 2RS (1) the Angle R2S 110°. 30′, (2) the Angle 2RS 45°. 10′ (3) the Angle 2SR 24°. 20′ the Complement of the other two to 180°, & (4) the Stationary distance 2R 176 feet whereby you may find the other Sides as before.

How to protract or lay down upon Paper or Parchment a distance taken according to the two last Directions by help of your Protractor or line of Chords.

Having made Observations in the field by the Theodolite or Circumferentor you are to note down the quantities of these

veral Lines & Angles observed in the field in a Book or Paper so that they may be ready at hand when you come to Protraction; & this is the usual Way.

Suppose it were required to draw upon Paper or Parchment the true Symmetry or Proportion of the Distance taken in the last.

First, upon your Paper draw a line at length as R2 then upon one end thereof as at R place the Center of your Protractor & lay the Meridian Line & F of the Protractor directly upon the line 2 R, the[n] (because the angle 2RS is 45° 10') against 45° 10 of your Protractor make a Mark upon your Paper with your protracting-pin & draw the line RS. This done from any Scale take your Stationary distance R2 176 feet & set it from R to 2, then upon the Point 2 (because the Angle R2S contains 110° 30') place the Center of the Protractor & turn it about till the line R2 lies directly under 110° 30'. Then (at the Point & of the Protractor) make a Mark with your protracting-pin & through that Point draw the line 2S which will cut the Line RS in the point S. Then if you measure the length of the lines 2S & RS by the same Scale from whence you took 176 for the line 2R you shall find the line 2S to contain 303 & the line RS to contain 400 agreeing exactly with the former Work.